

Probabilistic forecast verification

Caio Coelho

Centro de Previsão de Tempo e Estudos Climáticos (CPTEC)

Instituto Nacional de Pesquisas Espaciais (INPE)

Plan of lecture

- **Examples of probabilistic forecasts and common verification practice**
- **How to construct a reliability diagram**
- **Exercise on Brier score, its decomposition and reliability diagram**
- **ROC: discrimination**
- **Exercises on ROC**

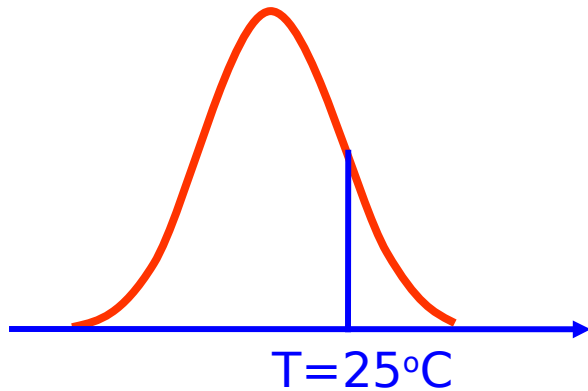
7th International Verification Methods Workshop

Tutorial on forecast verification methods

Berlin, Germany, 3-6 May 2017



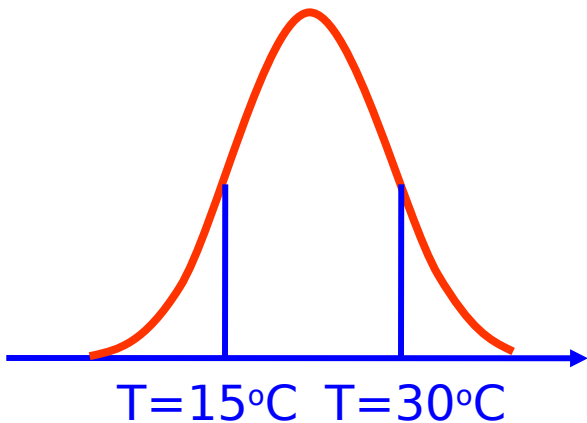
Examples of probabilistic forecasts: Temperature



F is a set of probabilities
for the discrete values of O

F: 0.4, 0.3, 0.5, 0.1, 0.6, 0.2

O: 1, 1, 0, 1, 0, 0



F is a probabilistic interval
of values for O (interval forecast)

F: 0.7, 0.6, 0.5, 0.8, 0.7, 0.5

O: 0, 1, 0, 1, 1, 0

Common verification practice:

- Compare forecast probability and occurrence (or non-occurrence) of event using a probabilistic score (e.g Brier score)
- Construct a reliability diagram

Forecast attributes assessed with the Brier score and reliability diagram

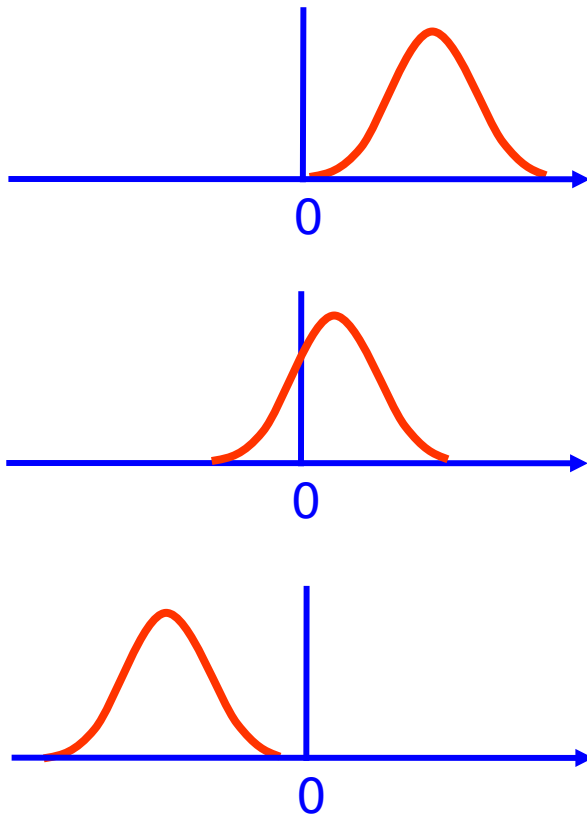
- Reliability: correspondence between forecast probabilities and observed relative frequency (e.g. an event must occur on 30% of the occasions that the 30% forecast probability was issued)
- Resolution: Conditioning of observed outcome on the forecasts
- Addresses the question: Does the frequency of occurrence of an event differ as the forecast probability changes?
- If the event occurs with the same relative frequency regardless of the forecast, the forecasts are said to have

Example of how to construct a reliability diagram

Sample of probability forecasts:

22 years x 3000 grid points = 66000 forecasts

How many times the event ($T > 0$) was forecast with probability p_i ?



| Forecast Prob.(p_i) | # Fcsts. N_i | “Perfect fcst.” OBS-Freq.(\bar{o}_i) | “Real fcst.” OBS-Freq(\bar{o}_i) |
|-------------------------|----------------|--|--------------------------------------|
| 100% | 8000 | 8000 (100%) | 7200 (90%) |
| 90% | 5000 | 4500 (90%) | 4000 (80%) |
| 80% | 4500 | 3600 (80%) | 3000 (66%) |
| | | | |
| | | | |
| | | | |
| 10% | 5500 | 550 (10%) | 800 (15%) |
| 0% | 7000 | 0 (0%) | 700 (10%) |

Courtesy: Francisco Doblas-Reyes

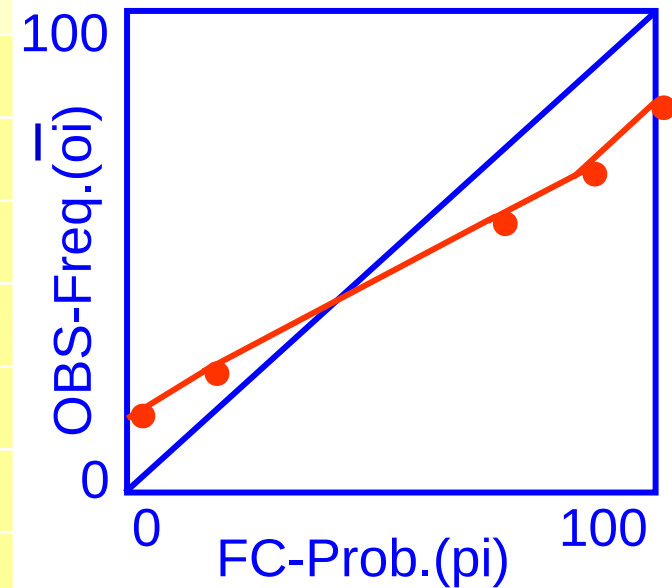
Example of how to construct a reliability diagram

Sample of probability forecasts:

22 years x 3000 grid points = 66000 forecasts

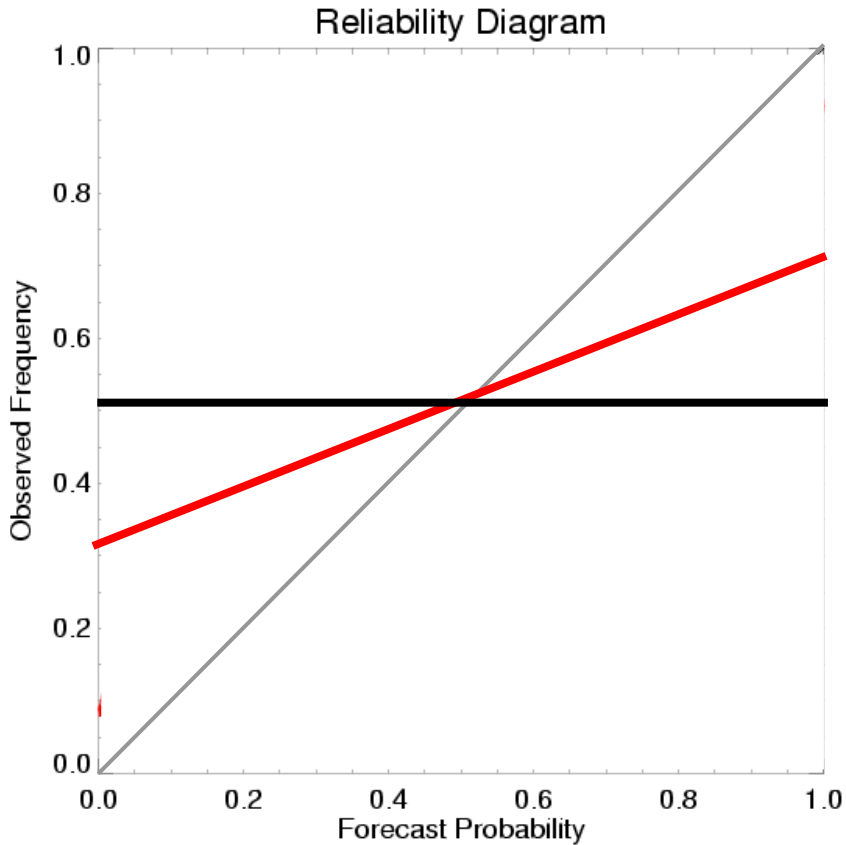
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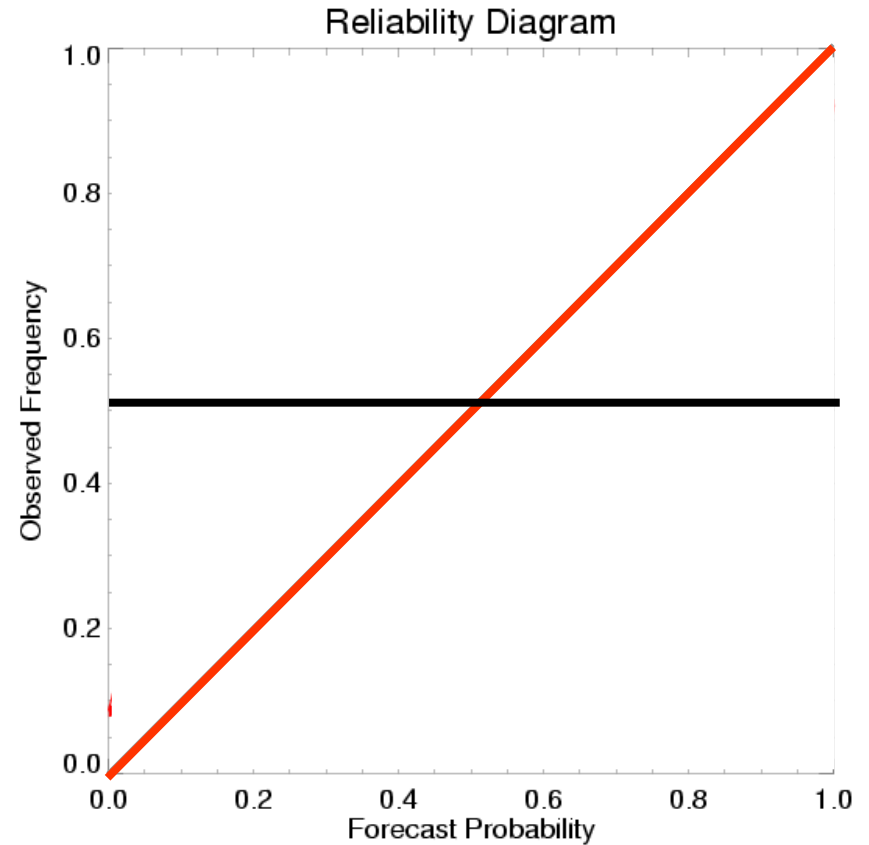


Reliability diagram

Over-confident forecasts,
with poor resolution

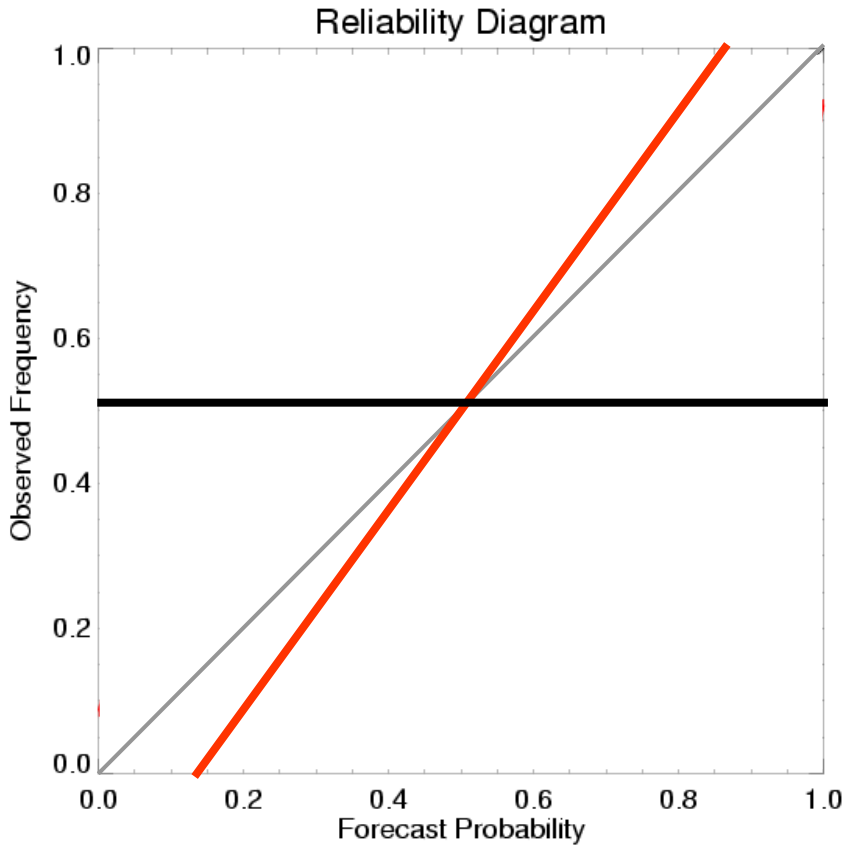


Perfect forecasts

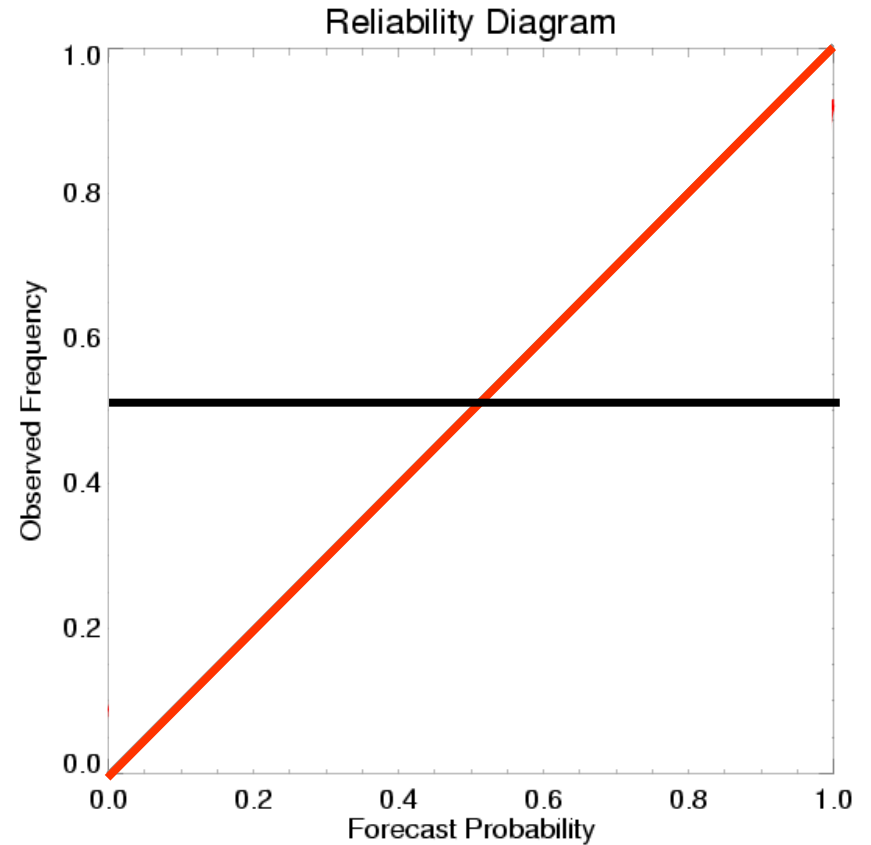


Reliability diagram

Under-confident forecasts,
with good resolution

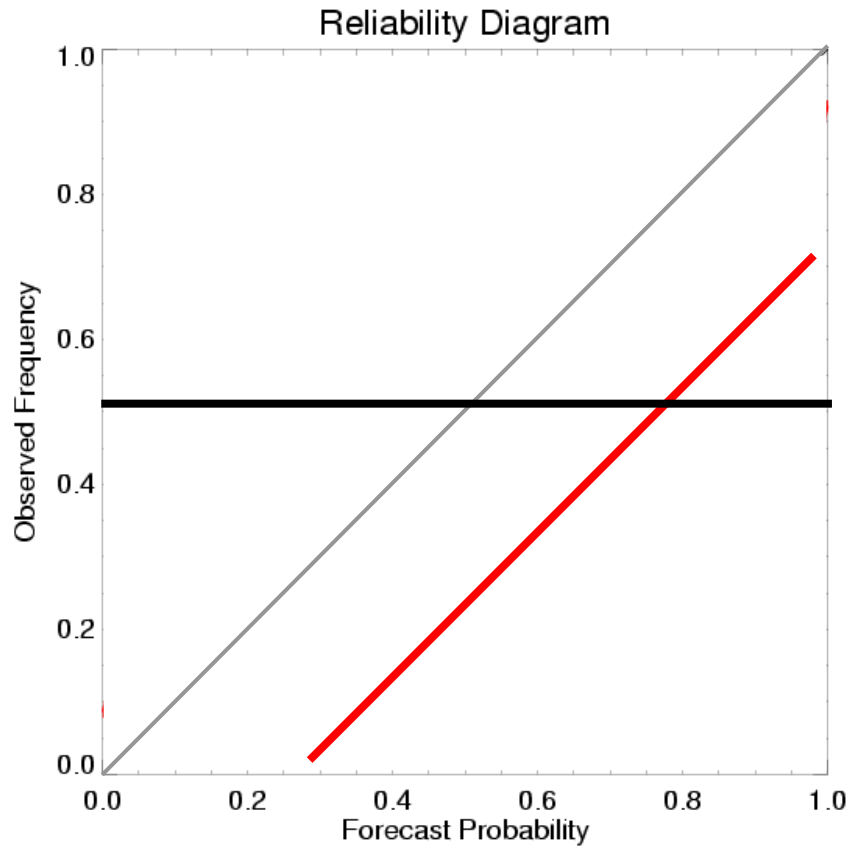


Perfect forecasts

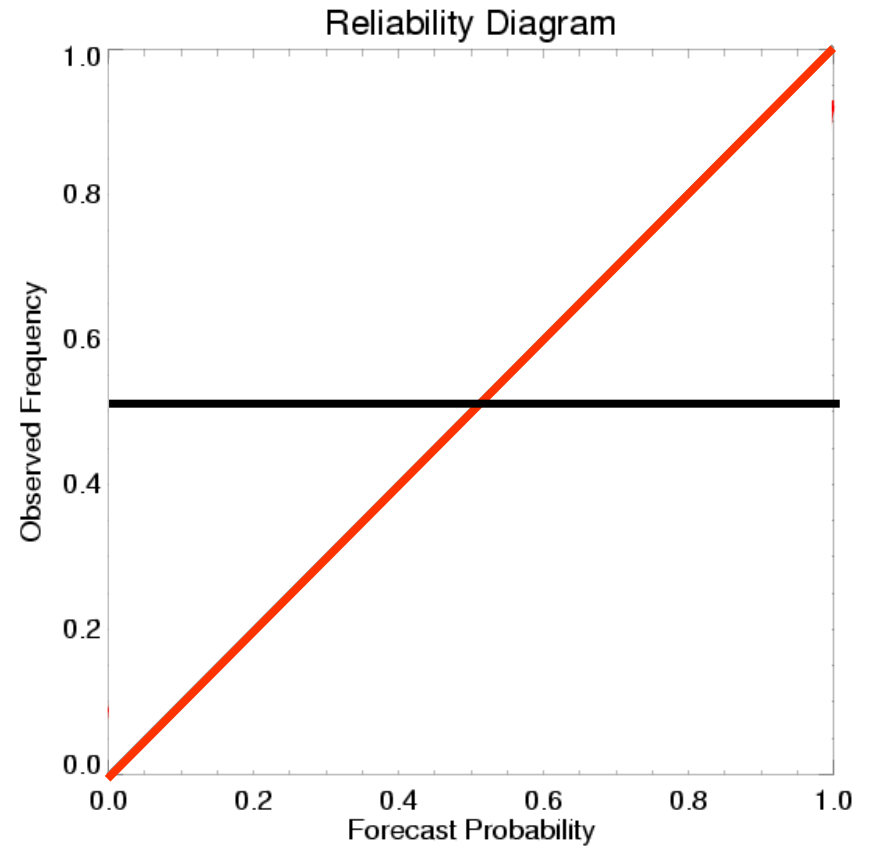


Reliability diagram

Over forecasting

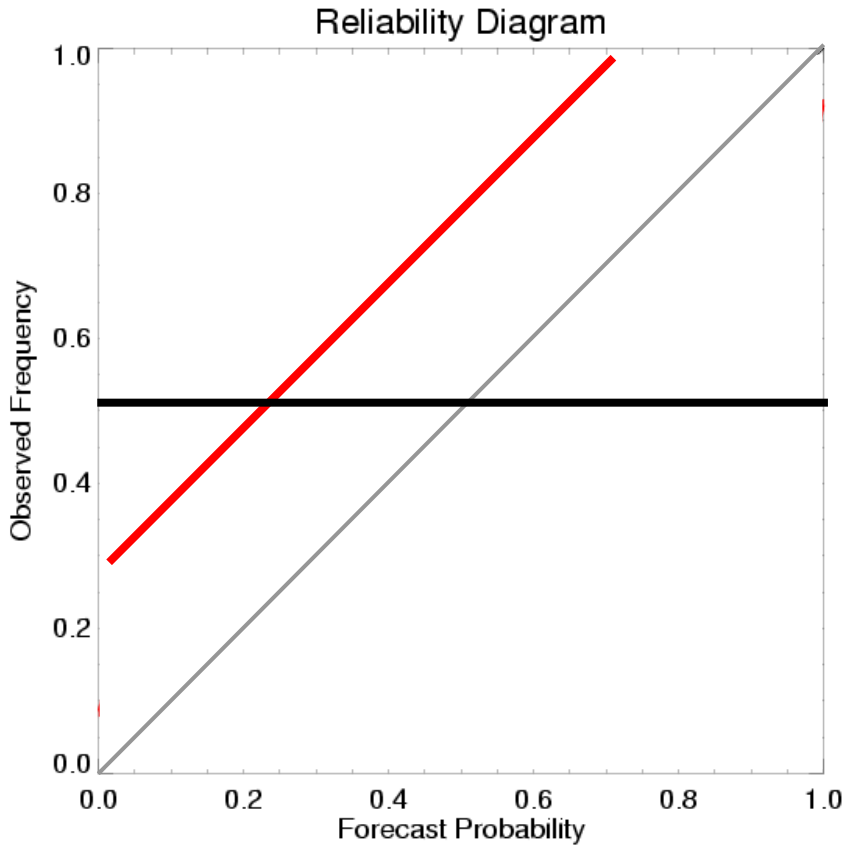


Perfect forecasts

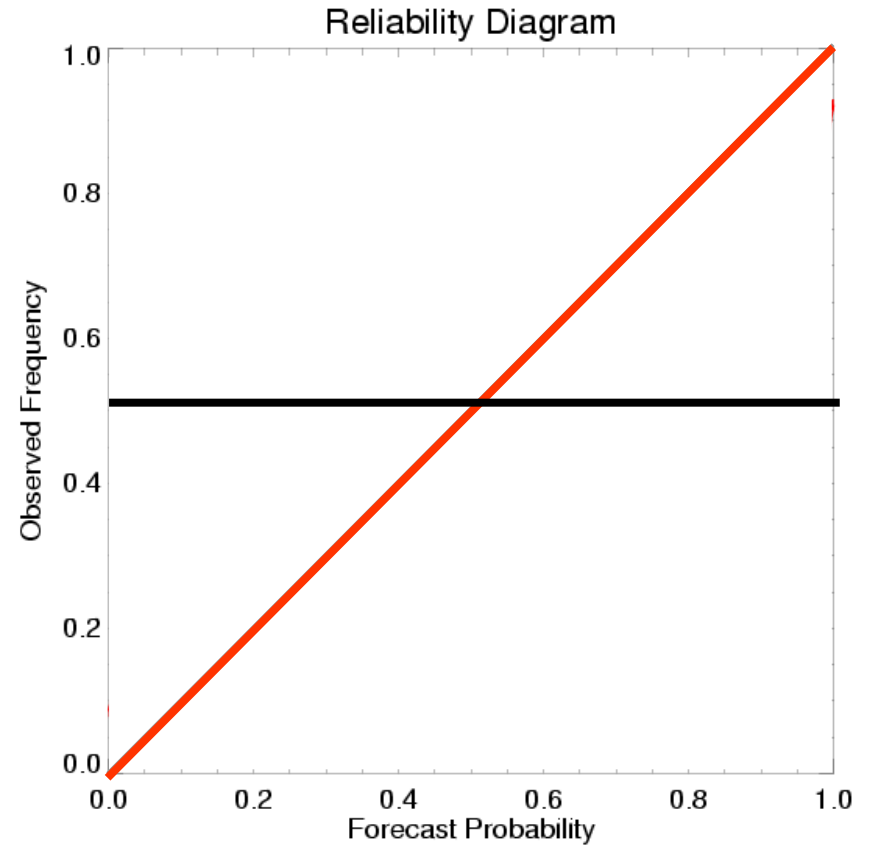


Reliability diagram

Under forecasting



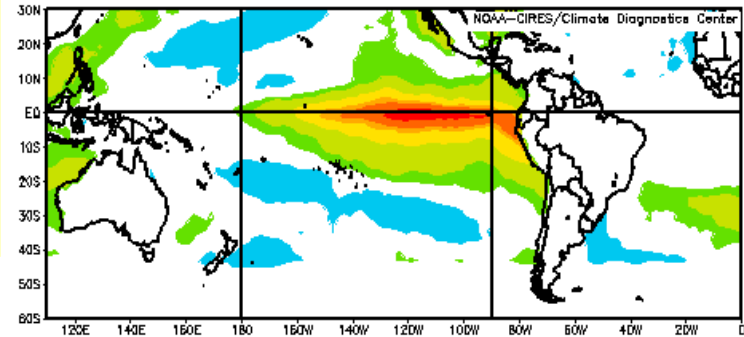
Perfect forecasts



Example: Equatorial Pacific SST

88 seasonal probability forecasts of binary SST anomalies at 56 grid points along the equatorial Pacific. Total of 4928 forecasts.

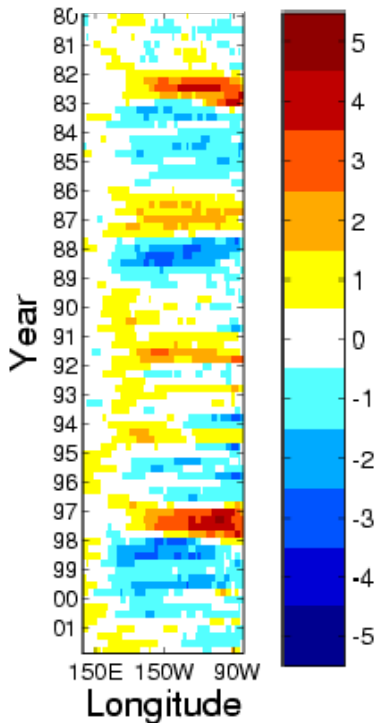
6-month lead forecasts for 4 start dates (F,M,A,N) valid for (Jul,Oct,Jan,Aug)



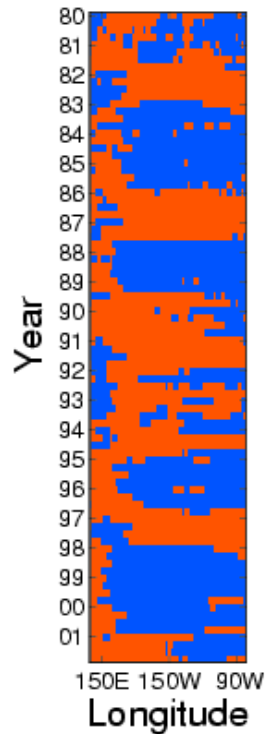
SST

$$o = (SST > 0) \quad f = \Pr(\hat{o})$$

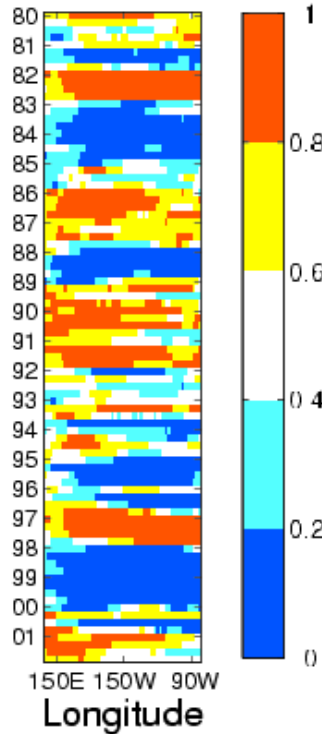
OBS



OBS



ENS



The probability forecasts were constructed by fitting Normal distributions to the ensemble mean forecasts from the 7 DEMETER coupled models, and then calculating the area under the normal density for SST anomalies greater than zero

SST₀ anomalies (°C)

Forecast probability

Exercise 1:

Read data file equatorialpacificsst.txt which contains forecast probabilities for the event Eq. Pac. SST>0 and the corresponding binary observations

```
data<-read.table("equatorialpacificsst.txt")
```

```
#1st column contains forecast probabilities  
probfcasts<-data[,1]
```

```
#2nd column contains binary observation  
binobs<-data[,2]
```

```
#Compute the climatological frequency of the event  
obars<-mean(binobs)
```

```
#Compute the Brier score for the climatological frequency  
#(i.e. the climatological forecast)  
bsclim<-mean((obars-binobs)^2)
```

```
#Compute the variance of binary observation  
var(binobs) * (length(binobs)-1)/length(binobs)
```

```
#Compute the uncertainty component of the Brier score  
obars*(1-obars)
```

```
#How does this compare with the Brier score computed  
#above? What can you conclude about the reliability and  
#resolution components of the Brier score for the  
#climatological forecast?
```

```
#Compute the Brier score for the SST prob. forecasts
```

```
#for the event SST>0
```

```
bs<-mean((probfcasts-binobs)^2)
```

```
#How does this compare with the Brier score for the  
#climatological forecast? What can you conclude about the  
#skill of these forecasts (i.e. which of the two are more  
#skillfull by looking at their Brier score values)?
```

```
#Compute the Brier skill score
```

```
bss <- 1-(bs/bsclim)
```

```
#How do you interpret the Brier skill score obtained  
#above? I.e. what can you conclude about the skill of the SST  
#prob. forecasts when compared to the climatological  
#forecast?
```

```
#Use the verification package to compute the Brier score and  
#its decomposition for the SST prob. forecasts for  
#the event SST>0  
library(verification)  
A<-verify(binobs,probfcsts, frcst.type="prob",obs.type="binary")  
summary(A)
```

```
#Note: Brier score – Baseline is the Brier score for the  
#reference climatological forecast  
#Skill Score is the Brier skill score  
#Reliability, resolution and uncertainty are the three  
#components of the Brier score decomposition
```

```
#What can be conclude about the quality of these forecasts  
#when compared with the climatological forecasts?
```

#Construct the reliability diagram for these forecasts using

#10 bins

```
nbins<-10
```

```
bk<-seq(0,1,1/nbins)
```

```
h<-hist(probfcsts,breaks=bk,plot=F)$counts
```

```
g<-hist(probfcsts[binobs==1],breaks=bk,plot=F)$counts
```

```
obari <- g/h
```

```
yi <- seq((1/nbins)/2,1,1/nbins)
```

```
par(pty='s',las=1)
```

```
reliability.plot(yi,obari,h,titl="10 bins",legend.names="")
```

```
abline(h=obari)
```

#What can you conclude about these forecasts by examining

#the feature of the reliability diagram curve?

```
# Compute reliability, resolution and uncertainty components
# of the Brier score
n<-length(probfcsts)
reliab <- sum(h*((yi-obari)^2), na.rm=TRUE)/n
resol <- sum(h*((obari-obar)^2), na.rm=TRUE)/n
uncert<-obbar*(1-obbar)
bs<-reliab-resol+uncert
```

#How does the results above compare with those obtained
#with the verify function?

Discrimination

- Conditioning of forecasts on observed outcomes
- Addresses the question: Does the forecast (probabilities) differ given different observed outcomes? Or, can the forecasts distinguish (discriminate or detect) an event from a non-event?

Example: Event (Positive SST anom. observed)

Non-event (Positive SST anom. not obs)

- If the forecast is the same regardless of the outcome, the forecasts cannot discriminate an *event* from a *non-event*
- Forecasts with no discrimination ability are useless because the forecasts are the same regardless of what happens

ROC: Relative operating characteristics

Measures discrimination (ability of forecasting system to detect the event of interest)

| Forecast | Observed | | |
|----------|----------|-----------------------|-----------|
| | Yes | No | Total |
| Yes | a (Hit) | b (False alarm) | a+b |
| No | c (Miss) | d (Correct rejection) | c+d |
| Total | a+c | b+d | a+b+c+d=n |

Hit rate= $a/(a+c)$

False alarm rate= $b/(b+d)$

ROC curve: plot of hit versus false-alarm rates for various prob. thresholds

Important points to remember

- The area under the ROC curve will tell us the probability of successfully discriminating an event from a non event. In other words, how different the forecast probabilities are for events and non events
- As events and non-events are binary (i.e have 2 possible outcomes) the probability of correctly discriminating (distinguishing) and event from a non-event by chance (guessing) is 50% and is represented by the area below the 45 degrees diagonal line in the ROC plot
- ROC is not sensitive to biases in the forecasts
- Forecast biases are diagnosed with the reliability diagram

Example: 3 category probabilistic forecasts

October NIÑO3 forecasts from five DEMETER models produced in the previous May, together with observed anomalies for the 20 years, 1982-2001. The observed NIÑO3 anomalies are indicated in column 2 and classified as El Niño (E), neutral (N), and La Niña (L).

- The forecasts are presented as probabilities based on a simple count of five DEMETER models.
- Sum of fcts probs E+N+L is 100%.
- Forecast probabilities for each category are assessed separately (i.e. each column of forecast probabilities for El Niño, Neutral and La Niña is assessed separately).
- Each fcst probability column is compared to obs column

| Year | Observed | | Probabilities of October NIÑO3 produced in the previous May | | |
|------|----------|---|---|-------------|-------------|
| | | | El Niño (E) | Neutral (N) | La Niña (L) |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

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Observed El Ninos: 5

Non Obs El Ninos: 15

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Prob_{≥0}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

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Prob_{≥0}

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$$H = 5/5 = 1$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

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Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥0}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$H = 5/5=1$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

$$F = 15/15=1$$

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Prob_{≥20}

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Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥20}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$H = 4/5 = 0.8$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

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| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥20}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$H = 4/5 = 0.8$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

$$F = 5/15 = 0.33$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥40}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥40}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$H = 4/5 = 0.8$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥40}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$H = 4/5 = 0.8$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

$$F = 3/15 = 0.2$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥60}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob \geq 60

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$H = 3/5 = 0.6$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥60}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$H = 3/5 = 0.6$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

$$F = 0/15 = 0$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥80}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥80}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$H = 3/5 = 0.6$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥80}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$H = 3/5 = 0.6$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

$$F = 0/15 = 0$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥100}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥100}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$H = 2/5 = 0.4$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob_{≥100}

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

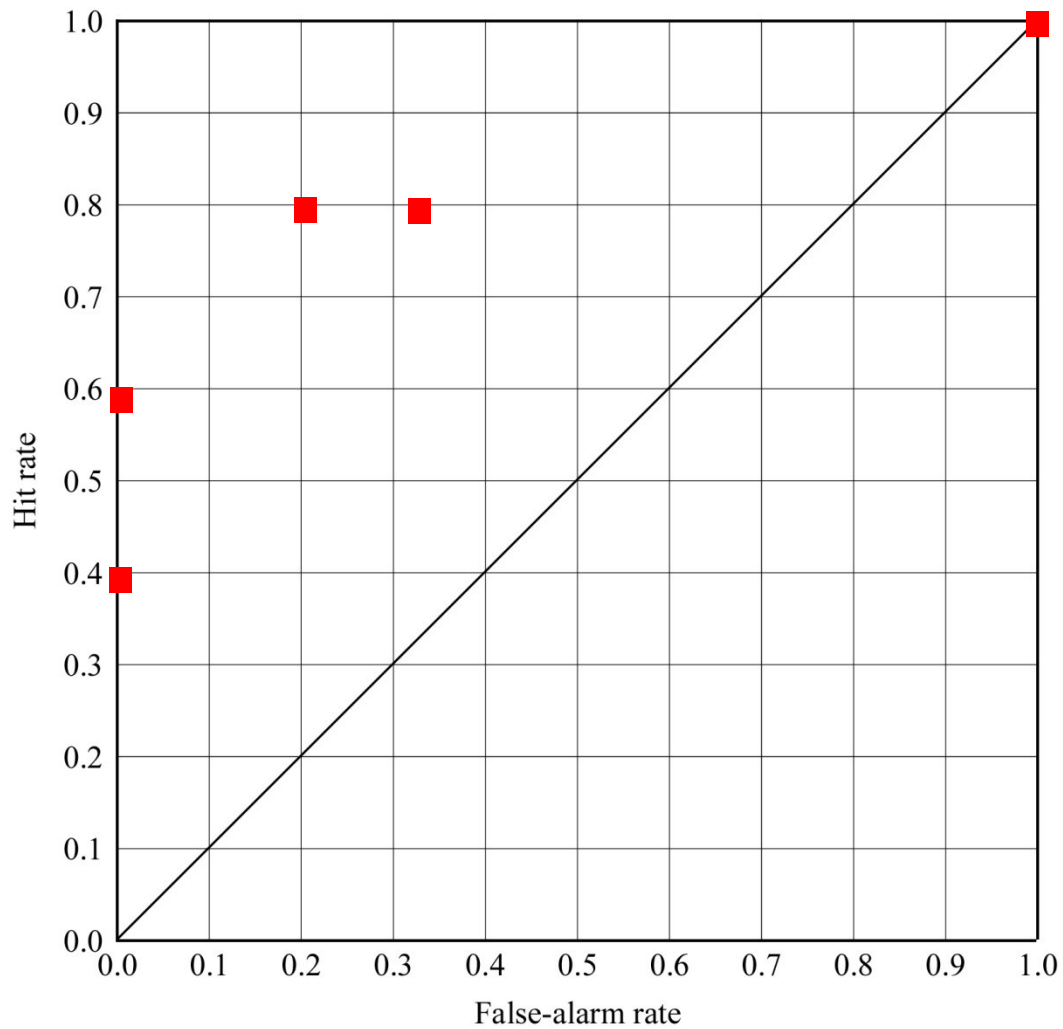
$$H = 2/5 = 0.4$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

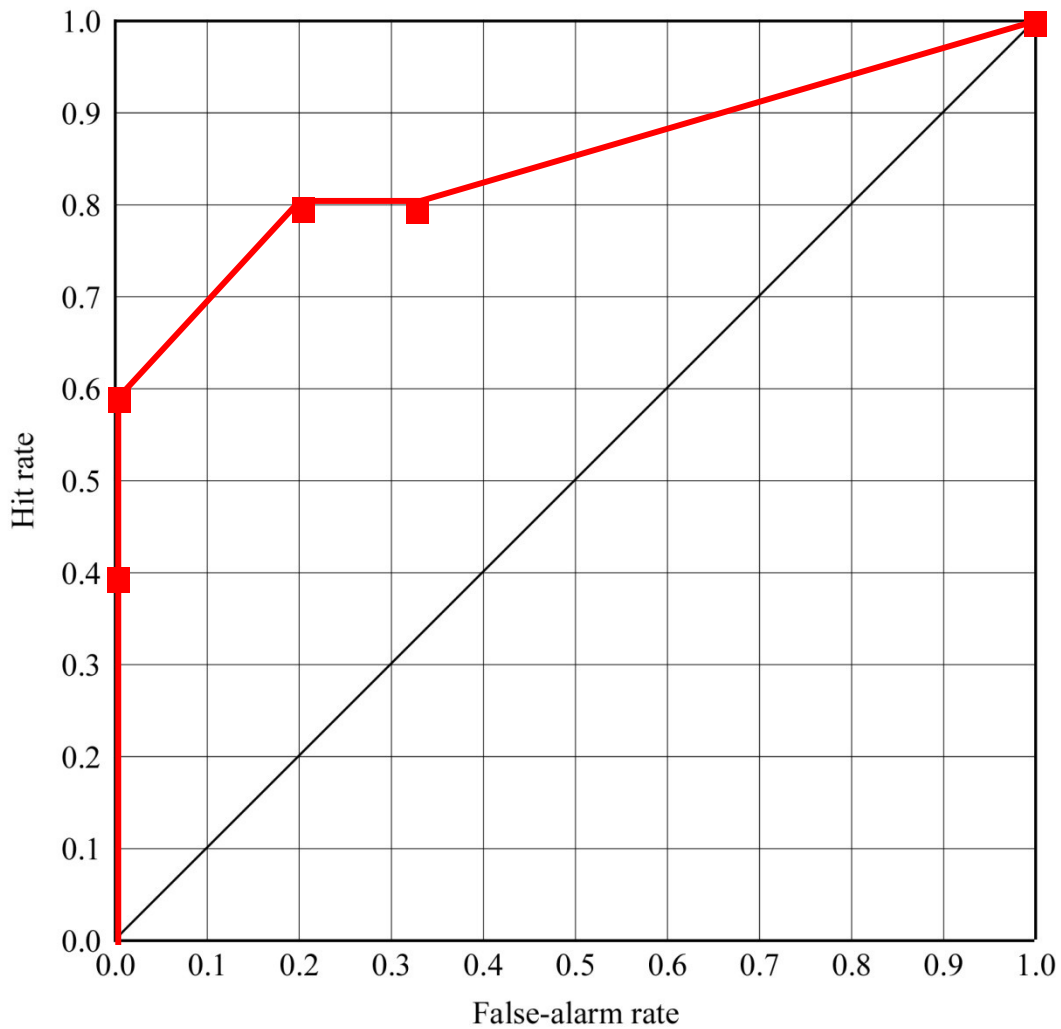
$$F = 0/15 = 0$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

| Warning | El Niño | | Neutral | | La Niña | |
|---------|---------|-----------|----------|----------|---------|------------|
| | H | F | H | F | H | F |
| ≥ 100% | 2/5=0.4 | 0/15=0 | 0/10=0 | 0/10=0 | 0/5=0 | 2/15=0.13 |
| ≥ 80% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 2/5=0.4 | 6/15=0.4 |
| ≥ 60% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 5/5=1 | 10/15=0.66 |
| ≥ 40% | 4/5=0.8 | 3/15=0.2 | 2/10=0.2 | 3/10=0.3 | 5/5=1 | 11/15=0.73 |
| ≥ 20% | 4/5=0.8 | 5/15=0.33 | 5/10=0.5 | 5/10=0.5 | 5/5=1 | 13/15=0.86 |
| ≥ 0% | 5/5=1 | 15/15=1 | 10/10=1 | 10/10=1 | 5/5=1 | 15/15=1 |

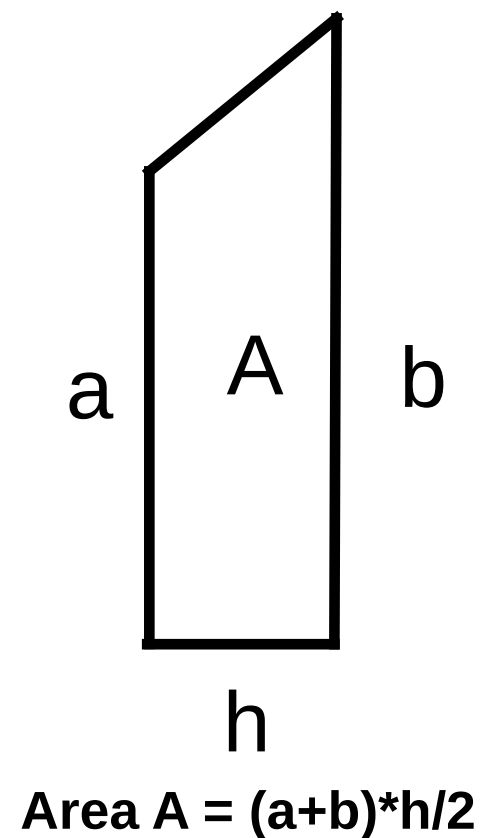
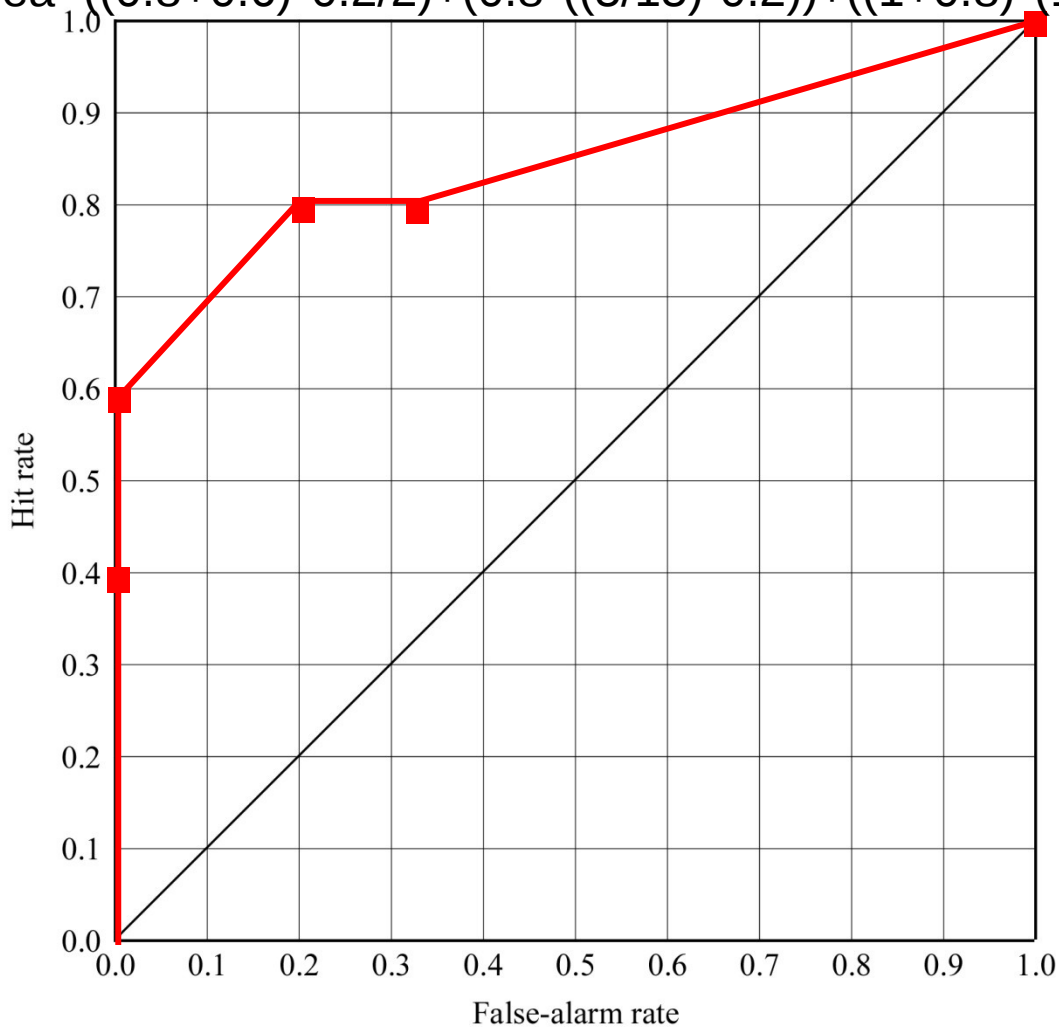


| Warning | El Niño | | Neutral | | La Niña | |
|---------|---------|-----------|----------|----------|---------|------------|
| | H | F | H | F | H | F |
| ≥ 100% | 2/5=0.4 | 0/15=0 | 0/10=0 | 0/10=0 | 0/5=0 | 2/15=0.13 |
| ≥ 80% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 2/5=0.4 | 6/15=0.4 |
| ≥ 60% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 5/5=1 | 10/15=0.66 |
| ≥ 40% | 4/5=0.8 | 3/15=0.2 | 2/10=0.2 | 3/10=0.3 | 5/5=1 | 11/15=0.73 |
| ≥ 20% | 4/5=0.8 | 5/15=0.33 | 5/10=0.5 | 5/10=0.5 | 5/5=1 | 13/15=0.86 |
| ≥ 0% | 5/5=1 | 15/15=1 | 10/10=1 | 10/10=1 | 5/5=1 | 15/15=1 |



| Warning | El Niño | | Neutral | | La Niña | |
|---------|---------|-----------|----------|----------|---------|------------|
| | H | F | H | F | H | F |
| ≥ 100% | 2/5=0.4 | 0/15=0 | 0/10=0 | 0/10=0 | 0/5=0 | 2/15=0.13 |
| ≥ 80% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 2/5=0.4 | 6/15=0.4 |
| ≥ 60% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 5/5=1 | 10/15=0.66 |
| ≥ 40% | 4/5=0.8 | 3/15=0.2 | 2/10=0.2 | 3/10=0.3 | 5/5=1 | 11/15=0.73 |
| ≥ 20% | 4/5=0.8 | 5/15=0.33 | 5/10=0.5 | 5/10=0.5 | 5/5=1 | 13/15=0.86 |
| ≥ 0% | 5/5=1 | 15/15=1 | 10/10=1 | 10/10=1 | 5/5=1 | 15/15=1 |

ROC area= $((0.8+0.6)*0.2/2)+(0.8*((5/15)-0.2))+((1+0.8)*(1-(5/15))/2)=0.85$



Observed Neutral: 10

Non Obs Neutral: 10

Prob_{≥0}

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

$$H = 10/10 = 1$$

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

$$F = 10/10 = 1$$

| Year | Observed | Probabilities of October NIÑO3 | | | |
|------|----------|--------------------------------|---------|---------|-----|
| | | El Niño | Neutral | La Niña | |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed Neutral: 10

Non Obs Neutral: 10

Prob_{≥20}

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

$$H = 5/10 = 0.5$$

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

$$F = 5/10 = 0.5$$

| | | Probabilities of October NIÑO3 | | | |
|------|----------|--------------------------------|---------|---------|---------|
| Year | Observed | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed Neutral: 10

Non Obs Neutral: 10

Prob_{≥40}

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

$$H = 2/10 = 0.2$$

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

$$F = 3/10 = 0.3$$

| Year | Observed | Probabilities of October NIÑO3 | | | |
|------|----------|--------------------------------|---------|---------|-----|
| | | El Niño | Neutral | La Niña | |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed Neutral: 10

Non Obs Neutral: 10

Prob \geq 60

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

$$H = 1/10 = 0.1$$

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

$$F = 0/10 = 0$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed Neutral: 10

Non Obs Neutral: 10

Prob \geq 80

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

$$H = 1/10 = 0.1$$

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

$$F = 0/10 = 0$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed Neutral: 10

Non Obs Neutral: 10

Prob_≥100

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

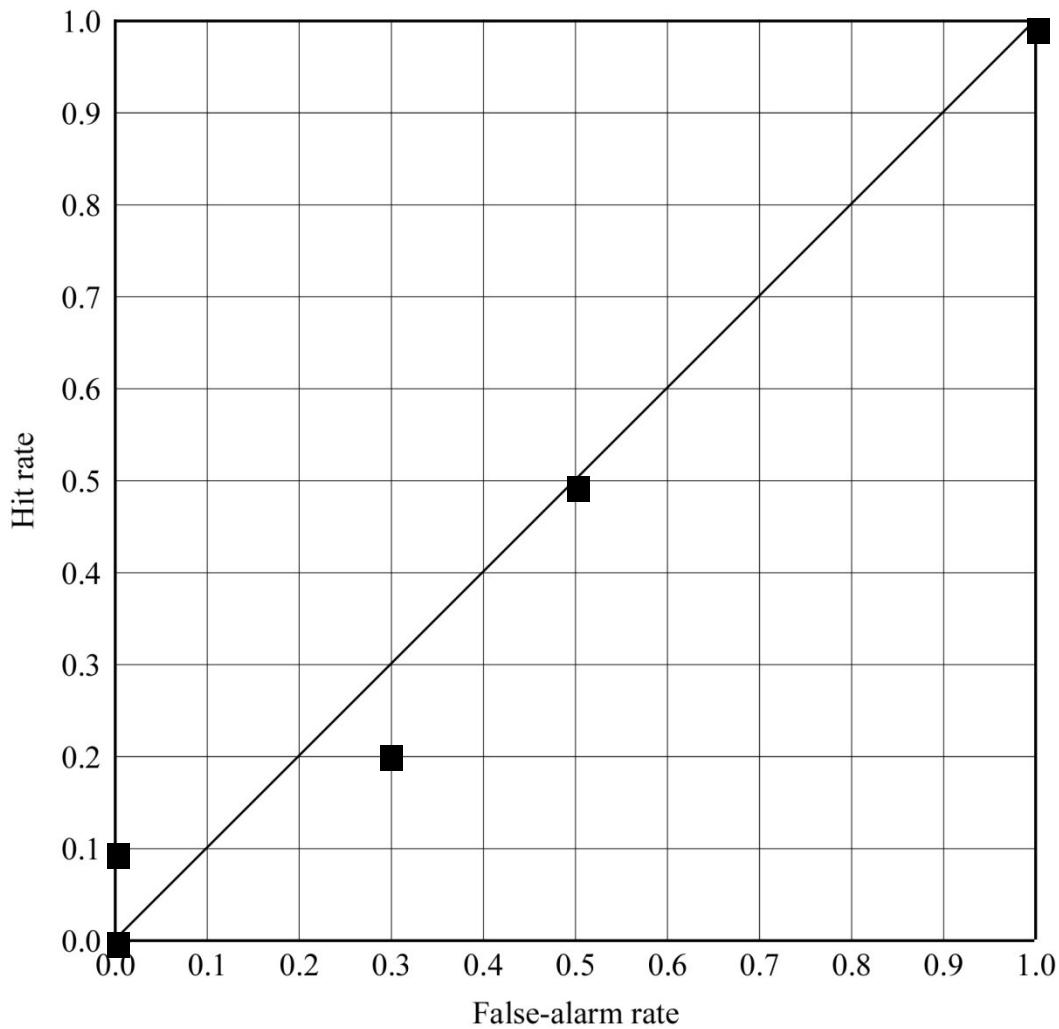
$$H = 0/10 = 0$$

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

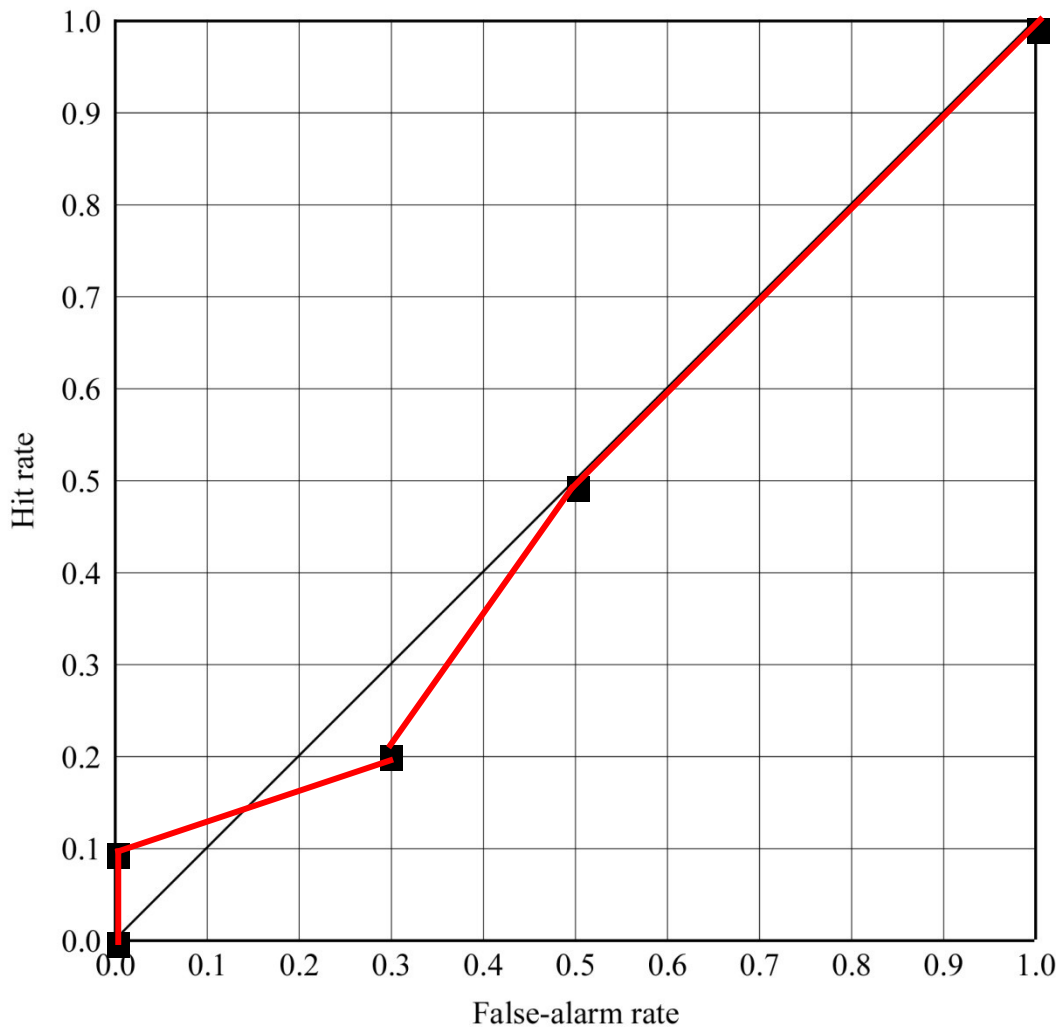
$$F = 0/10 = 0$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

| Warning | El Niño | | Neutral | | La Niña | |
|---------|---------|-----------|----------|----------|---------|------------|
| | H | F | H | F | H | F |
| ≥ 100% | 2/5=0.4 | 0/15=0 | 0/10=0 | 0/10=0 | 0/5=0 | 2/15=0.13 |
| ≥ 80% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 2/5=0.4 | 6/15=0.4 |
| ≥ 60% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 5/5=1 | 10/15=0.66 |
| ≥ 40% | 4/5=0.8 | 3/15=0.2 | 2/10=0.2 | 3/10=0.3 | 5/5=1 | 11/15=0.73 |
| ≥ 20% | 4/5=0.8 | 5/15=0.33 | 5/10=0.5 | 5/10=0.5 | 5/5=1 | 13/15=0.86 |
| ≥ 0% | 5/5=1 | 15/15=1 | 10/10=1 | 10/10=1 | 5/5=1 | 15/15=1 |

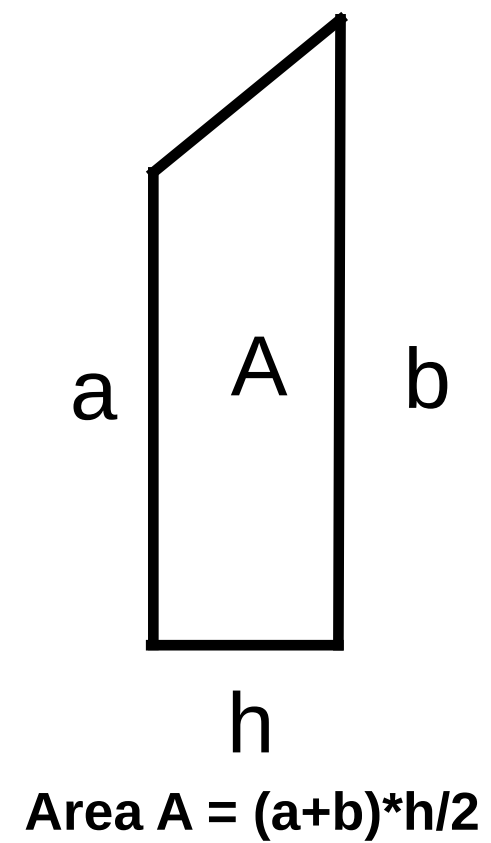
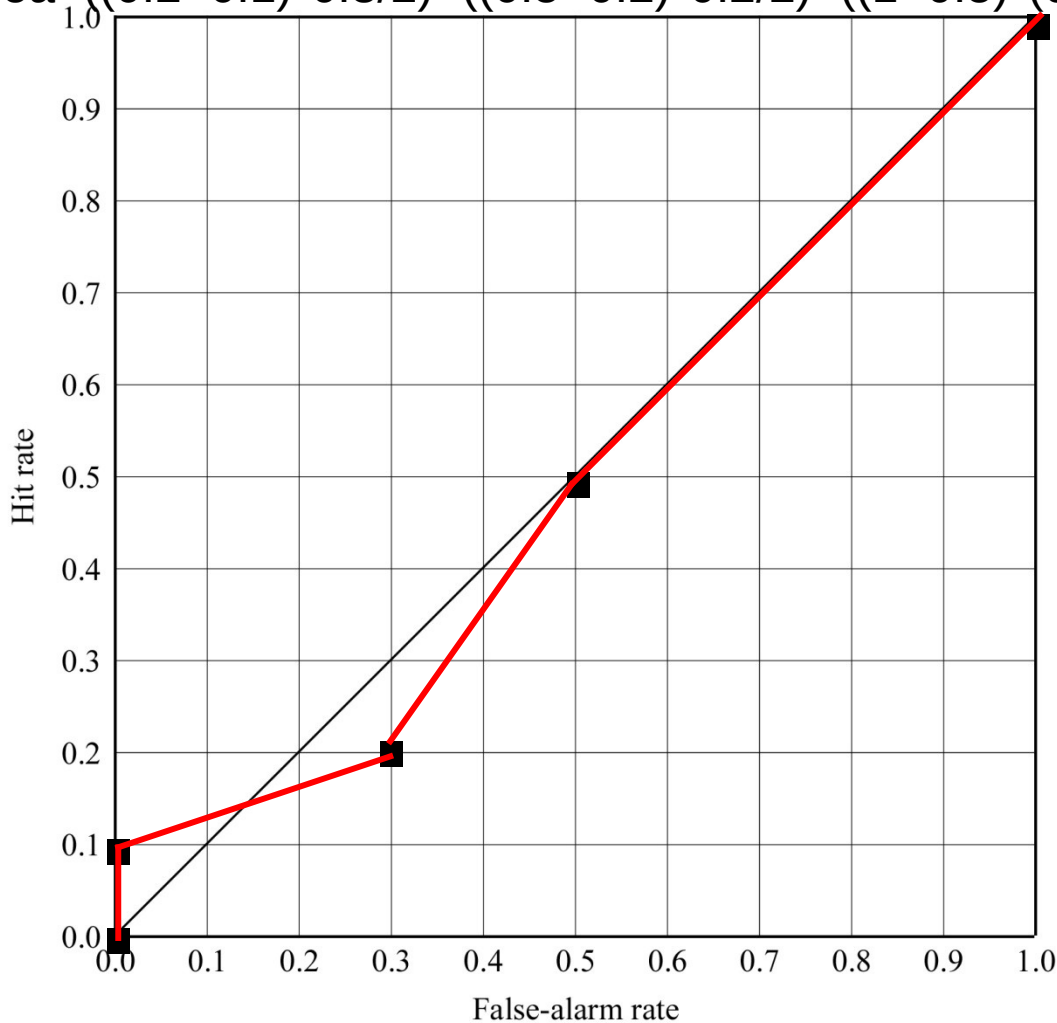


| Warning | El Niño | | Neutral | | La Niña | |
|---------|---------|-----------|----------|----------|---------|------------|
| | H | F | H | F | H | F |
| ≥ 100% | 2/5=0.4 | 0/15=0 | 0/10=0 | 0/10=0 | 0/5=0 | 2/15=0.13 |
| ≥ 80% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 2/5=0.4 | 6/15=0.4 |
| ≥ 60% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 5/5=1 | 10/15=0.66 |
| ≥ 40% | 4/5=0.8 | 3/15=0.2 | 2/10=0.2 | 3/10=0.3 | 5/5=1 | 11/15=0.73 |
| ≥ 20% | 4/5=0.8 | 5/15=0.33 | 5/10=0.5 | 5/10=0.5 | 5/5=1 | 13/15=0.86 |
| ≥ 0% | 5/5=1 | 15/15=1 | 10/10=1 | 10/10=1 | 5/5=1 | 15/15=1 |



| Warning | El Niño | | Neutral | | La Niña | |
|---------|---------|-----------|----------|----------|---------|------------|
| | H | F | H | F | H | F |
| ≥ 100% | 2/5=0.4 | 0/15=0 | 0/10=0 | 0/10=0 | 0/5=0 | 2/15=0.13 |
| ≥ 80% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 2/5=0.4 | 6/15=0.4 |
| ≥ 60% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 5/5=1 | 10/15=0.66 |
| ≥ 40% | 4/5=0.8 | 3/15=0.2 | 2/10=0.2 | 3/10=0.3 | 5/5=1 | 11/15=0.73 |
| ≥ 20% | 4/5=0.8 | 5/15=0.33 | 5/10=0.5 | 5/10=0.5 | 5/5=1 | 13/15=0.86 |
| ≥ 0% | 5/5=1 | 15/15=1 | 10/10=1 | 10/10=1 | 5/5=1 | 15/15=1 |

ROC area = $((0.2+0.1)*0.3/2) + ((0.5+0.2)*0.2/2) + ((1+0.5)*(0.5/2)) = 0.49$



Observed La Nina: 5

Non Obs La Nina: 15

Prob_{≥0}

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

$$H = 5/5 = 1$$

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

$$F = 15/15 = 1$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed La Nina: 5

Non Obs La Nina: 15

Prob_{≥20}

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

$$H = 5/5 = 1$$

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

$$F = 13/15 = 0.86$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed La Nina: 5

Non Obs La Nina: 15

Prob_{≥40}

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

$$H = 5/5 = 1$$

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

$$F = 11/15 = 0.73$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed La Nina: 5

Non Obs La Nina: 15

Prob_{≥60}

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

$$H = 5/5 = 1$$

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

$$F = 10/15 = 0.66$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed La Nina: 5

Non Obs La Nina: 15

Prob \geq 80

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

$$H = 2/5 = 0.4$$

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

$$F = 6/15 = 0.4$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

Observed La Nina: 5

Non Obs La Nina: 15

Prob_{≥100}

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

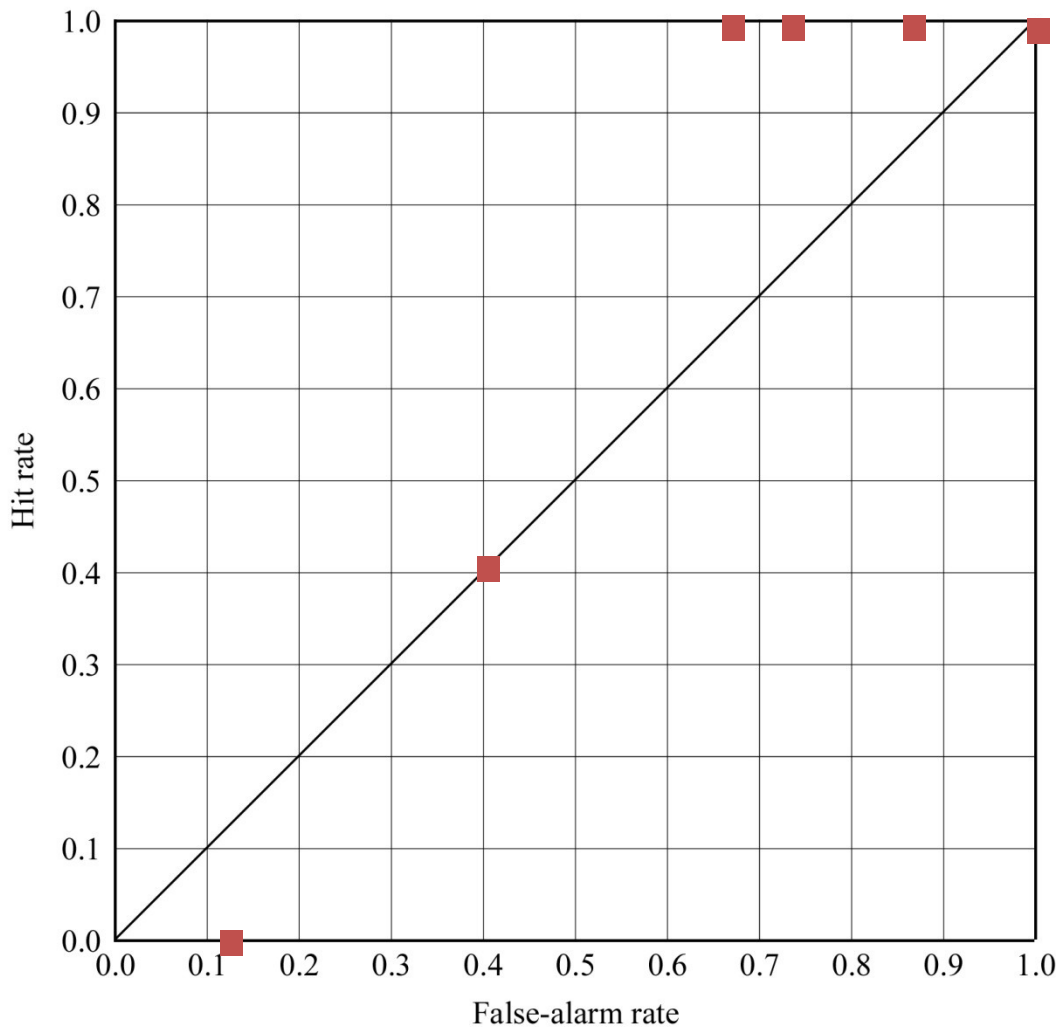
$$H = 0/5 = 0$$

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

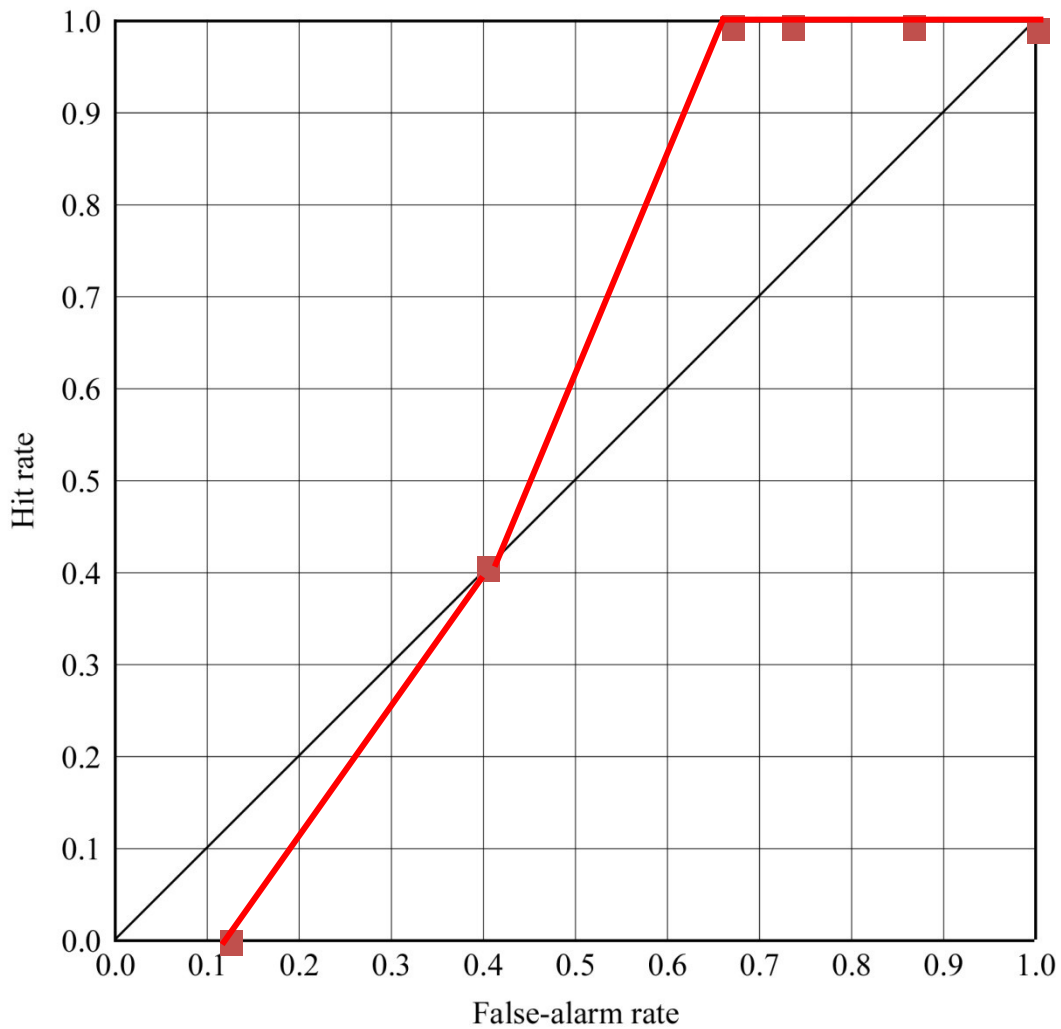
$$F = 2/15 = 0.13$$

| Year | Observed | | Probabilities of October NIÑO3 | | |
|------|----------|---|--------------------------------|---------|---------|
| | | | El Niño | Neutral | La Niña |
| 1981 | -0.23 | N | 0 | 40 | 60 |
| 1982 | 2.07 | E | 100 | 0 | 0 |
| 1983 | -0.21 | N | 0 | 0 | 100 |
| 1984 | -0.84 | L | 0 | 20 | 80 |
| 1985 | -0.82 | L | 0 | 20 | 80 |
| 1986 | 0.55 | E | 0 | 0 | 100 |
| 1987 | 1.28 | E | 80 | 0 | 20 |
| 1988 | -1.62 | L | 0 | 40 | 60 |
| 1989 | -0.41 | N | 20 | 0 | 80 |
| 1990 | -0.10 | N | 40 | 20 | 40 |
| 1991 | 0.62 | E | 40 | 0 | 60 |
| 1992 | -0.33 | N | 40 | 0 | 60 |
| 1993 | 0.24 | N | 40 | 0 | 60 |
| 1994 | 0.47 | N | 0 | 20 | 80 |
| 1995 | -0.86 | L | 0 | 40 | 60 |
| 1996 | -0.49 | N | 0 | 20 | 80 |
| 1997 | 3.02 | E | 100 | 0 | 0 |
| 1998 | -0.71 | N | 0 | 80 | 20 |
| 1999 | -1.09 | L | 0 | 40 | 60 |
| 2000 | -0.54 | N | 20 | 0 | 80 |

| Warning | El Niño | | Neutral | | La Niña | |
|---------|---------|-----------|----------|----------|---------|------------|
| | H | F | H | F | H | F |
| ≥ 100% | 2/5=0.4 | 0/15=0 | 0/10=0 | 0/10=0 | 0/5=0 | 2/15=0.13 |
| ≥ 80% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 2/5=0.4 | 6/15=0.4 |
| ≥ 60% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 5/5=1 | 10/15=0.66 |
| ≥ 40% | 4/5=0.8 | 3/15=0.2 | 2/10=0.2 | 3/10=0.3 | 5/5=1 | 11/15=0.73 |
| ≥ 20% | 4/5=0.8 | 5/15=0.33 | 5/10=0.5 | 5/10=0.5 | 5/5=1 | 13/15=0.86 |
| ≥ 0% | 5/5=1 | 15/15=1 | 10/10=1 | 10/10=1 | 5/5=1 | 15/15=1 |

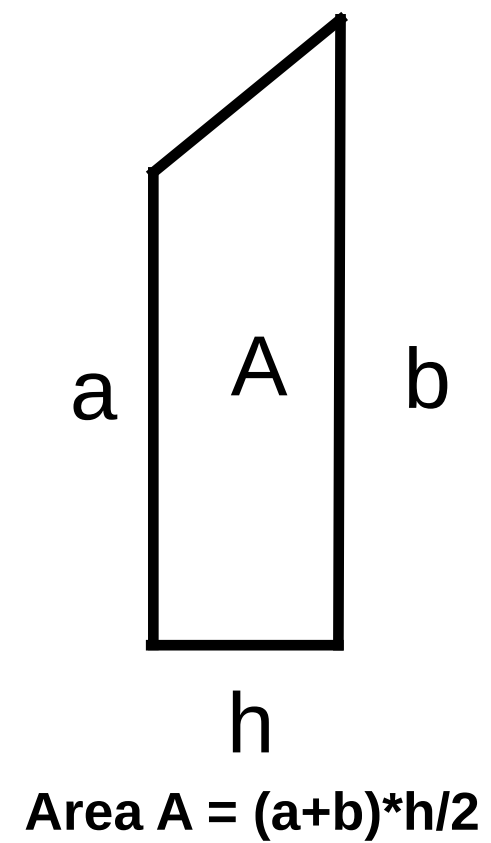
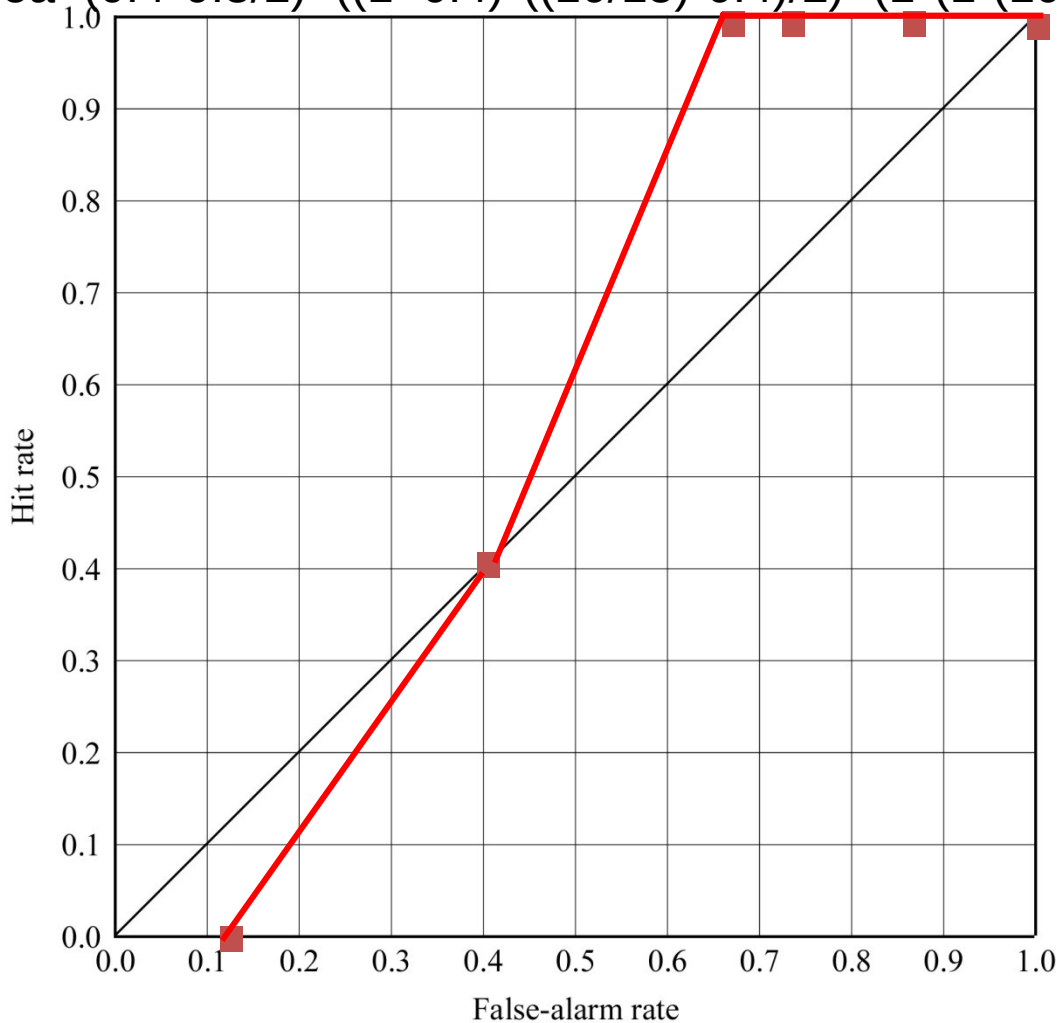


| Warning | El Niño | | Neutral | | La Niña | |
|---------|---------|-----------|----------|----------|---------|------------|
| | H | F | H | F | H | F |
| ≥ 100% | 2/5=0.4 | 0/15=0 | 0/10=0 | 0/10=0 | 0/5=0 | 2/15=0.13 |
| ≥ 80% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 2/5=0.4 | 6/15=0.4 |
| ≥ 60% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 5/5=1 | 10/15=0.66 |
| ≥ 40% | 4/5=0.8 | 3/15=0.2 | 2/10=0.2 | 3/10=0.3 | 5/5=1 | 11/15=0.73 |
| ≥ 20% | 4/5=0.8 | 5/15=0.33 | 5/10=0.5 | 5/10=0.5 | 5/5=1 | 13/15=0.86 |
| ≥ 0% | 5/5=1 | 15/15=1 | 10/10=1 | 10/10=1 | 5/5=1 | 15/15=1 |



| Warning | El Niño | | Neutral | | La Niña | |
|---------|---------|-----------|----------|----------|---------|------------|
| | H | F | H | F | H | F |
| ≥ 100% | 2/5=0.4 | 0/15=0 | 0/10=0 | 0/10=0 | 0/5=0 | 2/15=0.13 |
| ≥ 80% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 2/5=0.4 | 6/15=0.4 |
| ≥ 60% | 3/5=0.6 | 0/15=0 | 1/10=0.1 | 0/10=0 | 5/5=1 | 10/15=0.66 |
| ≥ 40% | 4/5=0.8 | 3/15=0.2 | 2/10=0.2 | 3/10=0.3 | 5/5=1 | 11/15=0.73 |
| ≥ 20% | 4/5=0.8 | 5/15=0.33 | 5/10=0.5 | 5/10=0.5 | 5/5=1 | 13/15=0.86 |
| ≥ 0% | 5/5=1 | 15/15=1 | 10/10=1 | 10/10=1 | 5/5=1 | 15/15=1 |

ROC area = $(0.4 * 0.3 / 2) + ((1 + 0.4) * ((10/15) - 0.4) / 2) + (1 * (1 - (10/15))) = 0.58$



Exercise 2:

Read data file equatorialpacificsst.txt which contains forecast probabilities for the event Eq. Pac. SST>0 and the corresponding binary observations

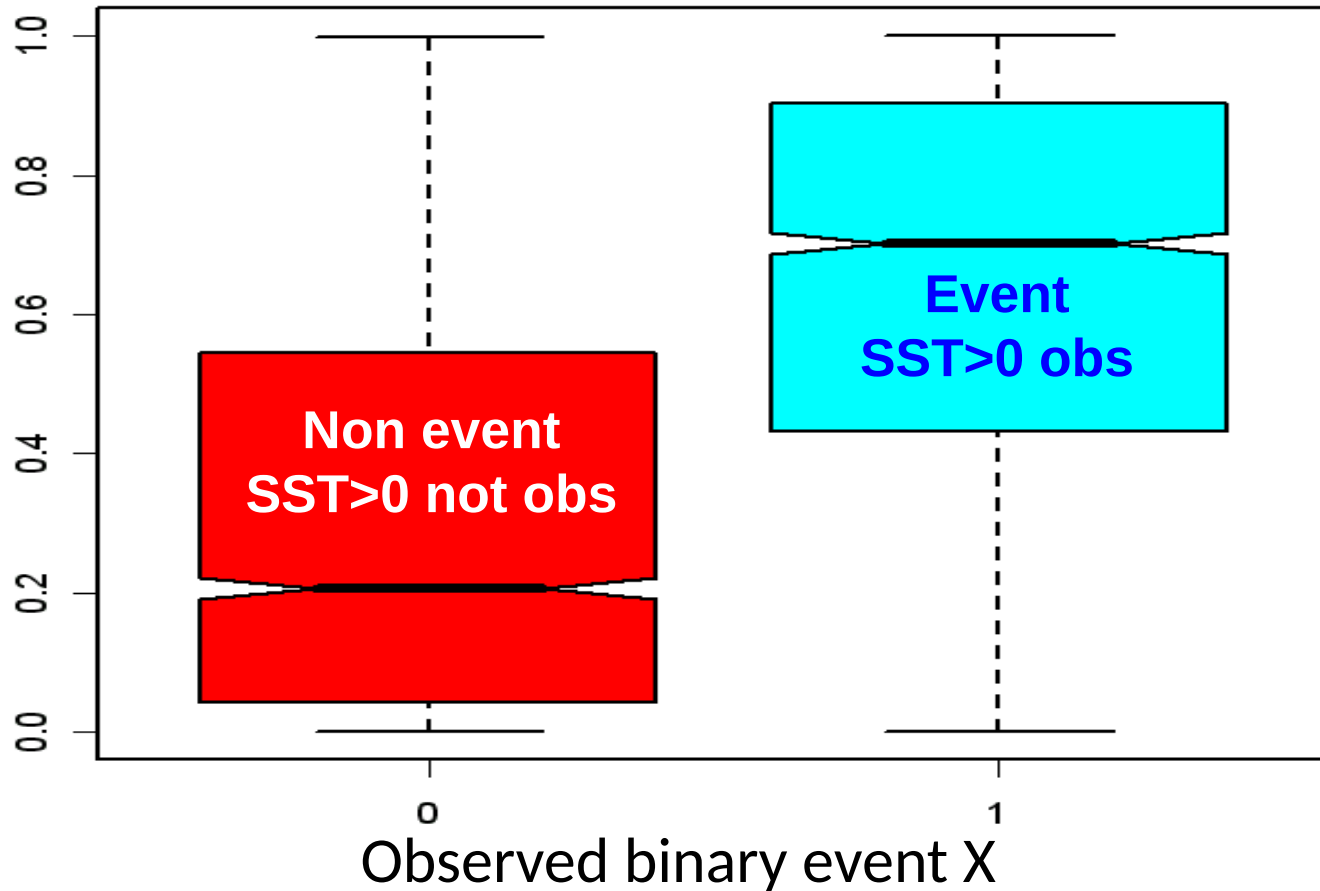
```
data<-read.table("equatorialpacificsst.txt")
```

#1st column contains forecast probabilities

#2nd column contains binary observations

Prob. forecasts conditioned/stratified

Forecast **on observations**
probability $\Pr(\text{SST}>0)$



- Forecasts do differ given different outcomes
- Forecast system has discrimination (distinguish eve

Reproducing the previous plot

1) Stratify forecast probabilities p (1st column of data) on observed (1) and not observed (0) binary events (2nd column of data)

`d1` #object containing strat of p on not observed event

```
> d1<-data[data[,2]==0,1]
```

`d2` #object containing strat of p on observed event

```
> d2<-data[data[,2]==1,1]
```

2) Produce a boxplot using the command

```
> boxplot(d1,d2,col=c(2,5),notch=T,names=c(0,1))
```

```
# extract only forecast/obs pairs with p >=0.9
p<-0.9
# forecast events
f<-data[data[,1]>=p,]
a<-sum(f[,2]==1) #forecast and observed (hit)
b<-sum(f[,2]==0) #forecast and not observed (false alarm)
# not forecast events
g<-data[data[,1]<p,]
c<-sum(g[,2]==1) #not forecast and observed (miss)
d<-sum(g[,2]==0) #not fcst and not obs (correct rejection)
n<-a+b+c+d
hr<-a/(a+c)
far<-b/(b+d)
```

```
#Plot first point of the ROC curve
par(pty='s',las=1)
plot(far,hr,type="p",pch=16,xlim=c(0,1),ylim=c(0,1),xlab="False alarm rate",ylab="Hit rate")
abline(0,1)
```

```
#repeat the same procedure for  $p \geq 0.8$ 
```

```
#extract only forecast/obs pairs with  $p \geq 0.8$ 
```

```
p<-0.8
```

```
# forecast events
```

```
f<-data[data[,1]>=p,]
```

```
a<-sum(f[,2]==1) #forecast and observed (hit)
```

```
b<-sum(f[,2]==0) #forecast and not observed (false alarm)
```

```
# not forecast events
```

```
g<-data[data[,1]<p,]
```

```
c<-sum(g[,2]==1) #not forecast and observed (miss)
```

```
d<-sum(g[,2]==0) #not fcst and not obs (correct rejection)
```

```
n<-a+b+c+d
```

```
hr<-a/(a+c)
```

```
far<-b/(b+d)
```

```
#Plot new point in the ROC curve  
points(far,hr,pch=16)
```

```
#repeat the same procedure for  $p \geq 0.7$ ,  $p \geq 0.6$ ,  $p \geq 0.5$ ,  
# $p \geq 0.4$ ,  $p \geq 0.3$ ,  $p \geq 0.2$  and  $p \geq 0.1$  adding the new points  
#in the ROC curve. Try later to do this using a for loop.
```

```
#The area below the curve that joins all points (the ROC  
#area) is a measure of discrimination.
```

```
#ROC area values equal 0.5 indicate no discrimination ability.
```

```
#ROC area values equal to 1 indicate perfect discrimination.
```

```
#ROC area values equal to 0 indicate perfectly bad
```

```
#discrimination.
```

```
#Constructing the empirical ROC curve
```

```
#find unique forecast probability values
```

```
p<-unique(data[,1])
```

```
#sort unique fcst prob values from largest to smallest
```

```
p<-rev(sort(p))
```

```
#define vectors to store hit and false-alarm rates
```

```
hr<-rep(NA,length(p)+2)
```

```
far<-rep(NA,length(p)+2)
```

```
#set first and last point in the ROC curve to (0,0) and (1,1)
```

```
hr[1]<-0
```

```
far[1]<-0
```

```
hr[length(p)+2]<-1
```

```
far[length(p)+2]<-1
```

```
#compute hit and false alarm rates for all fcst prob thresholds
for (i in 1:length(p)){
f<-data[data[,1]>=p[i],] #forecast events
a<-sum(f[,2]==1) #hit
b<-sum(f[,2]==0) #false alarm
g<-data[data[,1]<p[i],] # not forecast events
c<-sum(g[,2]==1) #miss
d<-sum(g[,2]==0) #correct rejection
hr[i+1]<-a/(a+c)
far[i+1]<-b/(b+d)
}
#plot empirical ROC curve
par(pty='s',las=1)
plot(far,hr,type="l",xlim=c(0,1),ylim=c(0,1),xlab="False alarm
rate",ylab="Hit rate")
abline(0,1)
```

```
#plot roc curve with verification package for comparison
x11()
roc.plot(data[,2],data[,1])

#compute area under empirical ROC curve
dif<-diff(far)
area<-sum(0.5 * (hr[1:((length(hr)-1))] + hr[2:length(hr)]) * dif)

#compute ROC area using the verification package
roc.area(data[,2],data[,1])

#The ROC skill score is defined as (2*ROC area)-1
#so that positive values indicate good discrimination skill
#and negative values indicate bad discrimination skill
rss<-2*area-1
```