A wavelet-based scale-separation verification approach to assess the added value of enhanced resolution models

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Motivation: enhanced resolution NWP systems produce precipitation forecast fields enriched of realistic small-scale details. Traditional verification approaches however fail to detect the added value of the enhanced resolution, possibly due to the higher variability and small timing and location displacements.

Aim: develop a scale-separation verification approach which enables to compare the performance (bias, error and skill) of coarse versus high resolution forecasts. We wish to extract the added value of the enhanced resolution!

Data: we test the new scale-separation verification on the MesoVICT (http://www.ral.ucar.edu/projects/icp/) case studies. We verify the Environment Canada GEM (Global Environmental Multi-scale) model, run in three configurations:

- GEMglb = global, ~ 33km
- GEMreg = LAM regional, ~ 15km
- GEMhres = LAM high-res, ~ 2.5km

against the VERA analysis ~ 8km.

Fig.s 2 to 7 show verification statistics evaluated for the MesoVICT core case study illustrated in Fig. 1.

Fig. 1. MesoVICT core case study

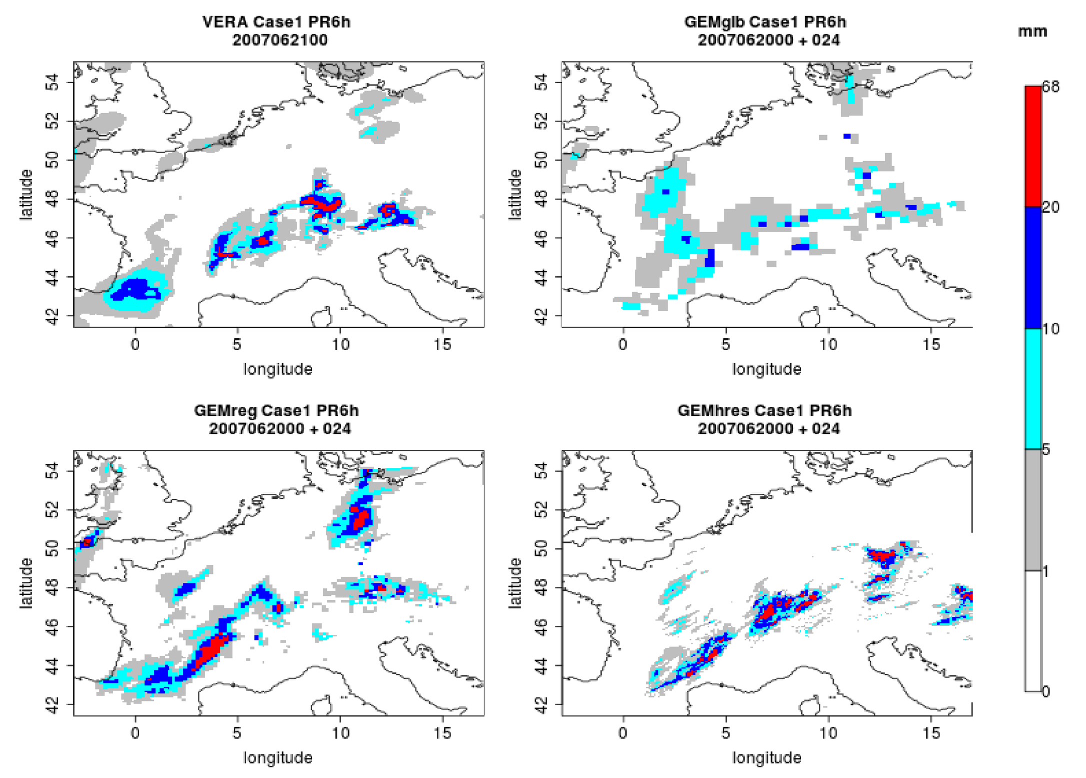


Fig. 2. The wavelet-based scale-separation MSE skill score

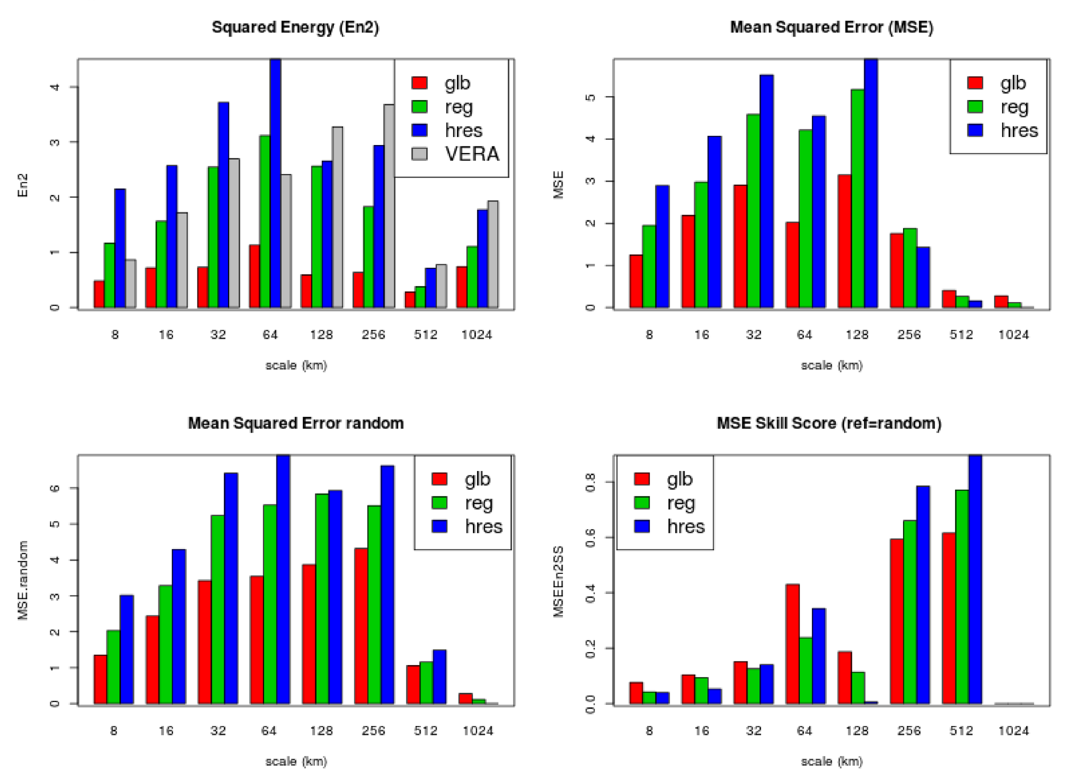


Fig. 3. MSE and energy for the scale components of the binary images

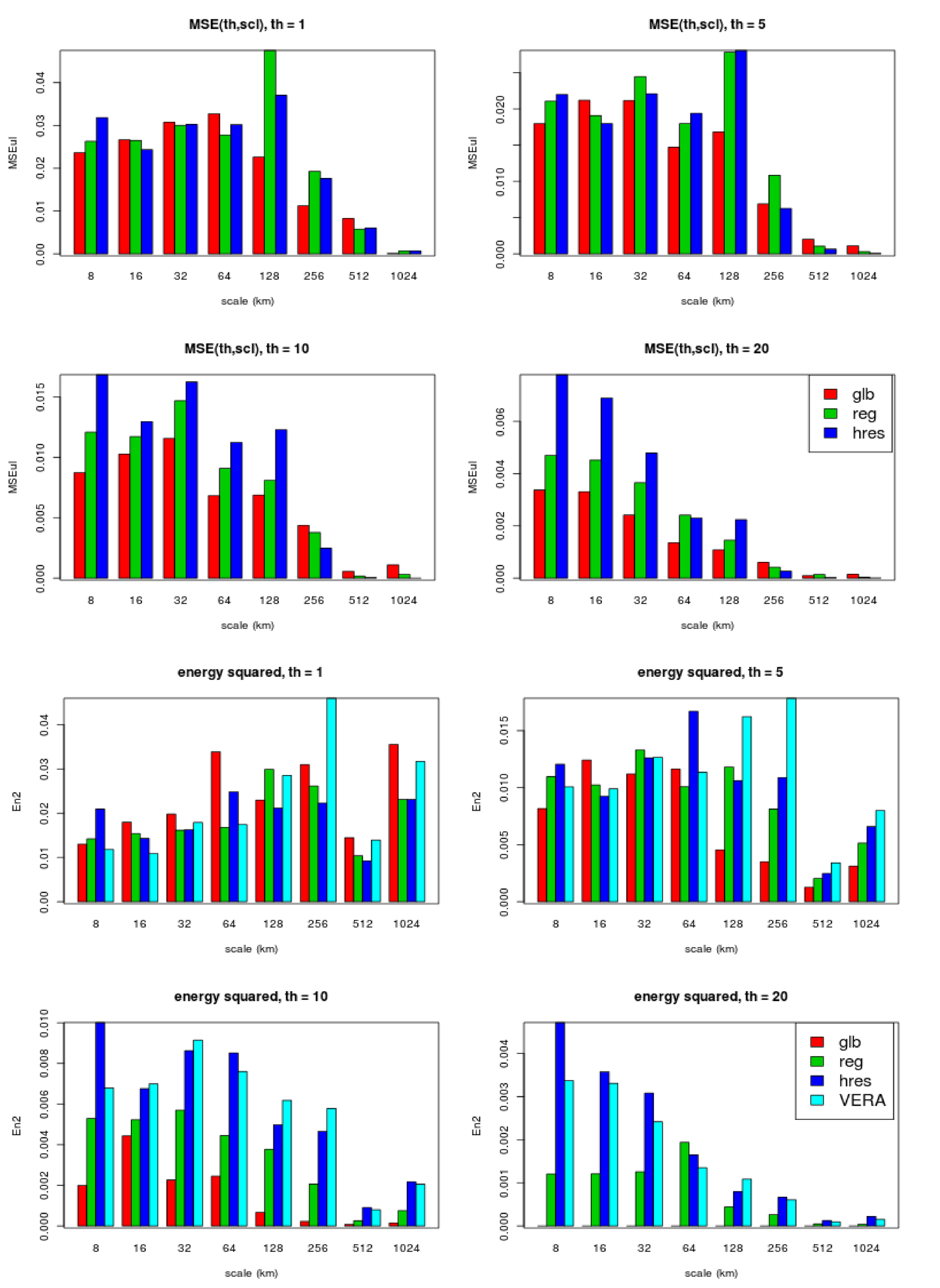


Fig. 4. Contingency table statistics

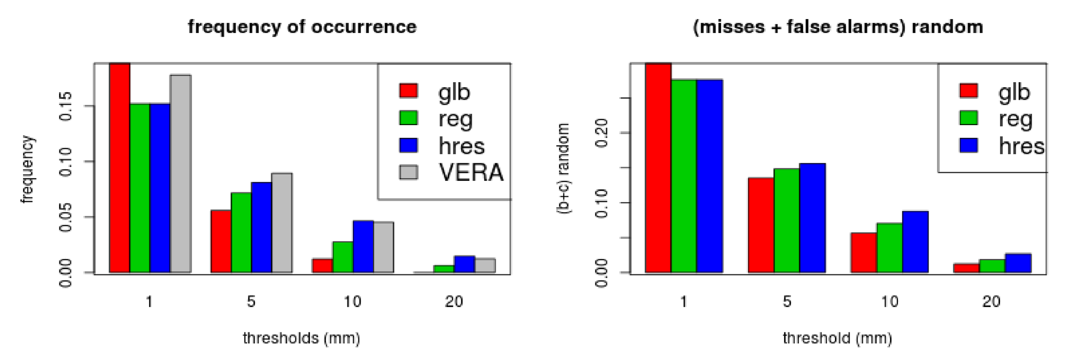


Fig. 5. Intensity-Scale skill score

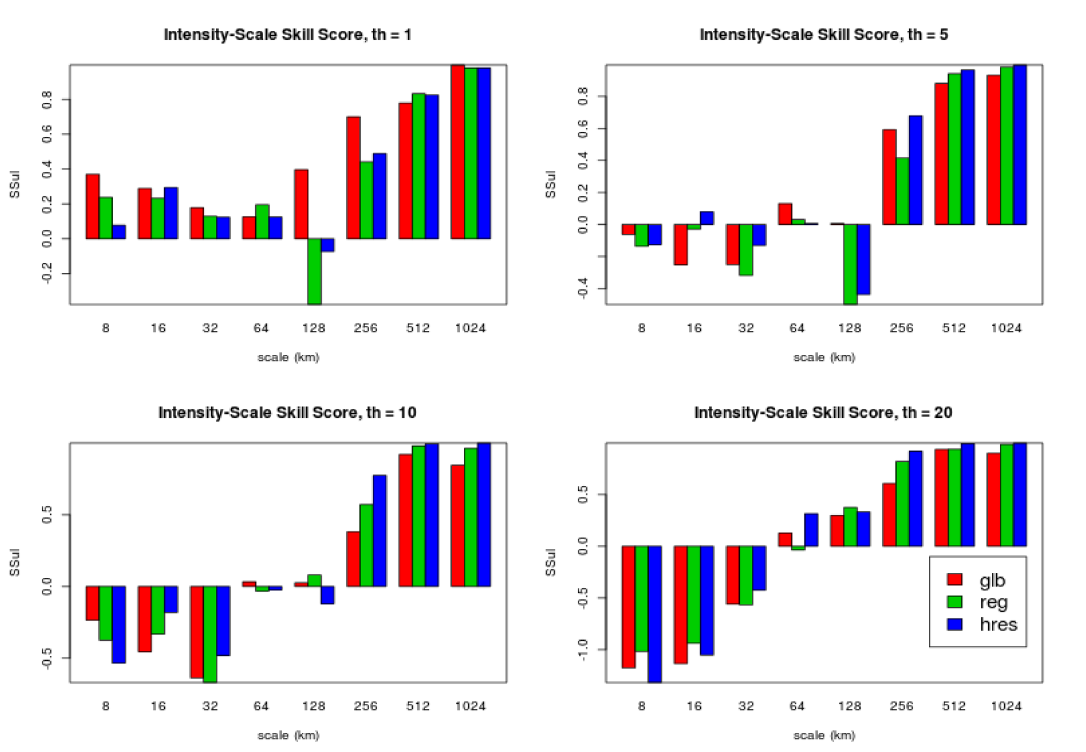
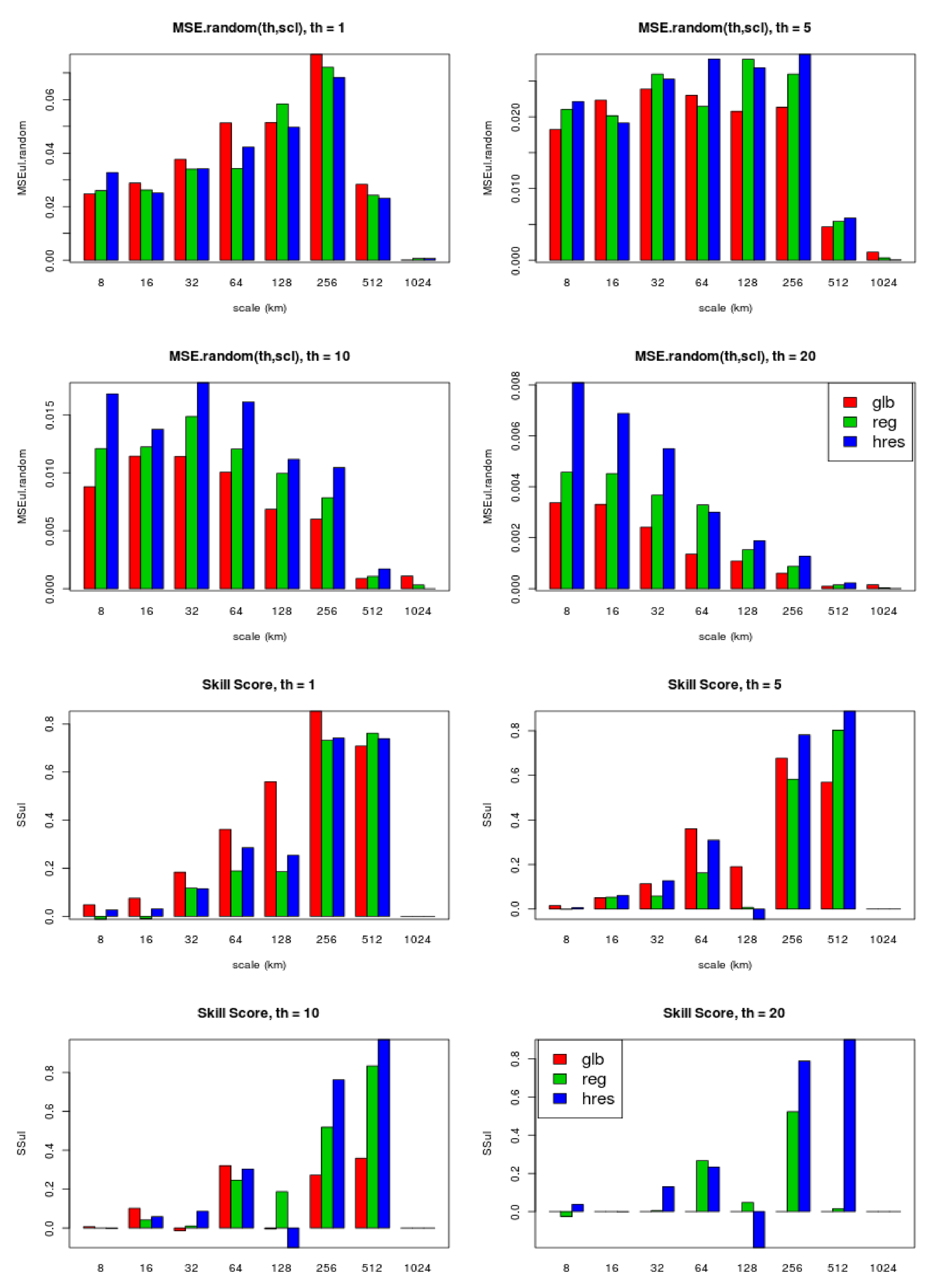
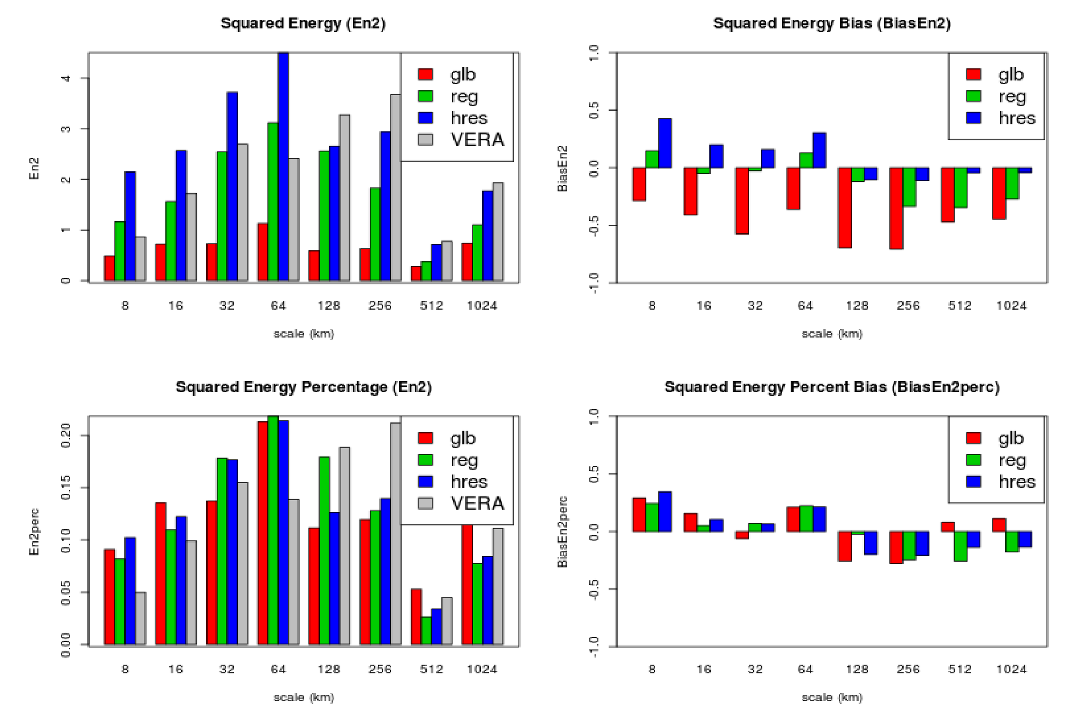


Fig. 6. *MSErandom* and skill score evaluated as in 4 and 5, but for the scale components of the binary images



The energy bias and percentage



12. The energy bias for each scale component is assessed by evaluating the relative difference of forecast and obs energies:

$$Bias_j^m(En^2)=\frac{En^2(Y_j)-En^2(X_j)}{En^2(Y_j)+En^2(X_j)} \quad \text{for } W_j^m$$

$$Bias_J^f(En^2)=(\bar{Y}^2-\bar{X}^2)/(\bar{Y}^2+\bar{X}^2) \quad \text{for } W_J^f$$

13. Due to the wavelet orthogonality:

$$En^2(X)=En^2(X_j)+En^2(\bar{X})$$

14. The forecast versus observed scale structure can be assessed by the relative difference (evaluated as in 12) of forecast and obs energy percentages:

$$En^{2m}_{j,percent}(X)=En^2(X_j)/En^2(X) \quad \text{for } W_j^m$$

$$En^{2f}_{J,percent}(X)=En^2(\bar{X})/En^2(X) \quad \text{for } W_J^f$$

The wavelet-based scale-separation MSE skill score

1. Forecast (Y) and obs (X) are decomposed into the sum of components on different scales by using 2D discrete Haar wavelet transforms:

$$X=\sum_{j=1}^{J} W_j^m(X)+W_J^f(X)=\sum_{j=1}^{J} X_j+\bar{X}$$

2. The Mean Squared Error (MSE) and the forecast and obs energy (En^2) are then evaluated for each scale component:

$$MSE_j^m=MSE(Y_j,X_j)=\overline{(Y_j-X_j)^2};\ En^2(X_j)=\overline{X_j^2} \quad \text{for } W_j^m$$

$$MSE_J^f=MSE(\bar{Y},\bar{X})=(\bar{Y}-\bar{X})^2;\ En^2(\bar{X})=\bar{X}^2 \quad \text{for } W_J^f$$

3. Note that:

$$MSE(Y,X)=(\bar{Y}-\bar{X})^2+s_Y^2+s_X^2-2s_Y s_X r_{Y,X}$$

$$\text{for } W_j^m \quad \bar{X}_j=0 \rightarrow s_X^2=En^2(X_j)$$

$$\text{for } W_J^f(X)=\bar{X} \rightarrow s_X^2=0$$

4. Therefore, the MSE for random forecast on each scale components is given by:

$$MSE^m_{j,random}=En^2(X_j)+En2(Y_j) \quad \text{for } W_j^m$$

$$MSE^f_{J,random}=(\bar{Y}-\bar{X})^2 \quad \text{for } W_J^f$$

5. The MSE skill score (with reference=random) is finally evaluated on each scale component as:

$$SS_j^m=1-MSE_j^m/MSE^m_{j,random} \quad \text{for } W_j^m$$

$$SS_J^f=0 \quad \text{for } W_J^f$$

Note 1: the reference forecast accounts for the forecast variability, therefore the skill score is suitable for comparing forecasts with different resolutions.

Note 2: *MSEj,random* is distributed across the scales in proportion to the energy (i.e. number of events and magnitude of the signal on each scale).

Improvements on the Intensity-Scale skill score

Casati et al (2004) Met.Apps. **11**
Casati (2010) Wea&For **25**

6. Intensity: Forecast and obs are transformed into binary images (*Yu,Xu*) by thresholding the precipitation intensity.

7. Scale: The binary forecast and obs are decomposed into the sum of components on different scales by using discrete Haar wavelet transforms.

8. For each threshold u and spatial scale j, the Mean Squared Error (*MSEu,j*) and energy (*En²u,j*) of the scale components of the binary images are evaluated (Fig. 3).

9. In virtue of the thresholding, the IS statistics are related to the contingency table entries (Fig. 4) as:

$$MSE_u=\frac{b+c}{n};\ En^2(Y_u)=\frac{a+b}{n};\ En^2(X_u)=\frac{a+c}{n}$$

10. the *MSEu,random* for random binary images can also be evaluated from the contingency table entries, in virtue of the Bayes theorem, as product of the marginals:

$$MSE_{u,random}=\frac{a+b}{n}\frac{b+d}{n}+\frac{a+c}{n}\frac{c+d}{n}=$$
$$=En^2(Y_u)(1-En^2(X_u))+En^2(X_u)(1-En^2(Y_u))$$

11. The IS skill score (Fig. 5) is then calculated as:

$$IS_{u,j}=1-MSE_{u,j}/MSE_{u,j,random}$$

where *MSEu,j,random* is the *MSEu,random* equally partitioned across the scales.

Issue: in real practice, the MSE for a random forecast is not equally partitioned across the scales.

Solution: the *MSEu,j,random* is calculated as in 4, and the IS skill score is then re-evaluated as in 5 (Fig. 6).

Result: the scale-separation MSE skill score evaluated without thresholding (Fig. 2) synthesize the IS skill scores evaluated for the thresholded binary images (Fig. 6). Similarly, En^2 bias and En^2% bias evaluated without thresholding synthesize the En^2 bias and En^2% bias evaluated for the thresholded binary images (not shown).

Conclusions

- The wavelet-based scale-separation statistics defined in equations 1-5 and 12-14 are informative on the forecast scale structure, bias, error and skill on different scale components, and are suitable for comparing models with different resolutions.
- Time series of these scale-dependent statistics are used to analyze the MesoVICT case studies.

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