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A Spatio-Temporal User-Centric Distance

Dominique Brunet*, David Sills* and Barbara Casati+
Meteorological Research Division
***Toronto & +Montréal, Canada**

Motivation

For **thunderstorms** and other **localized severe weather events**, *near misses* occur more frequently than actual damage to property or loss of life, but the consequences of actual events can be disastrous.

The **timing** and **location** of the forecast relative to the impacted persons is the most important aspect that needs to be predicted.

Potential users:
- Outdoor event planners (concerts, sports, festivals)
- Outdoor facility managers (campground, park)
- Air traffic controllers
- Civil authorities
- General public

The New Metric

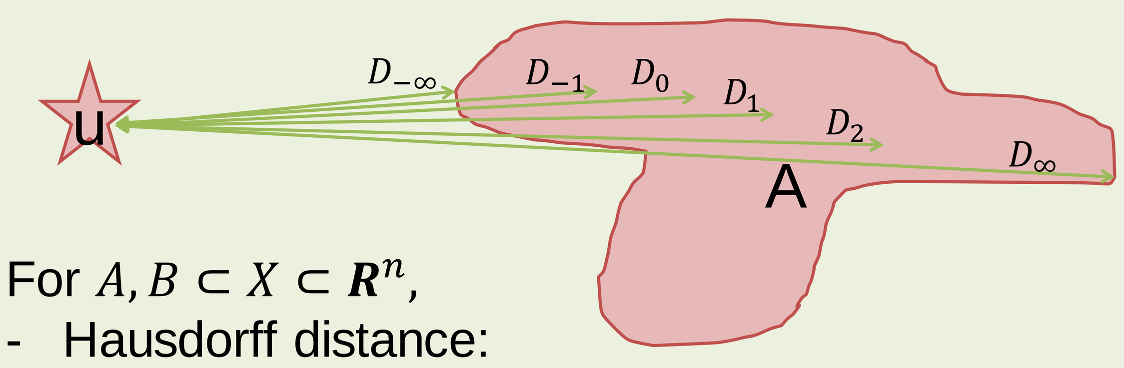
The Generalized Distance Transform (**GDT**) **[1]**:

User location — Geodesic distance

$$D_q(u,A) = \left(\frac{1}{|A|}\int_A d^q(u,a)\,da\right)^{\frac{1}{q}}.$$

Set of points — Size of A

Parameter q controls the influence of the points of A for the computation of the GDT:
$q \to -\infty$: minimal distance (distance transform)
$q = -1$: harmonic mean distance
$q \to 0$: geometric mean distance
$q = 1$: arithmetic mean distance
$q = 2$: root mean square distance
$q \to \infty$: maximum distance



For $A, B \subset X \subset \boldsymbol{R}^n$,
- Hausdorff distance:
  $H(A,B) = \max_{x\in X}|D_{-\infty}(x,A) - D_{-\infty}(x,B)|$
- Baddeley's delta-metric:
  $\Delta_p(A,B) = \left(\int_X |D_{-\infty}(x,A) - D_{-\infty}(x,B)|^p dx\right)^{\frac{1}{p}}$;
- Generalized **[1]**:
  $\Delta_{p,q}(A,B) = \left(\int_X |D_q(x,A) - D_q(x,B)|^p dx\right)^{\frac{1}{p}}$.

For $q \le 0, u \in A \Rightarrow D_q(u,A) = 0$.

For smaller q in magnitude, $D_q(u,A)$ is resilient to outliers.

$H, \Delta_p, \Delta_{p,q}$ are valid distance-metrics.

- Time-series of GDT vs time:

$$d_O(t) = D_q(u(t), O(t))$$
$$d_F(t) = D_q(u(t), F(t)).$$

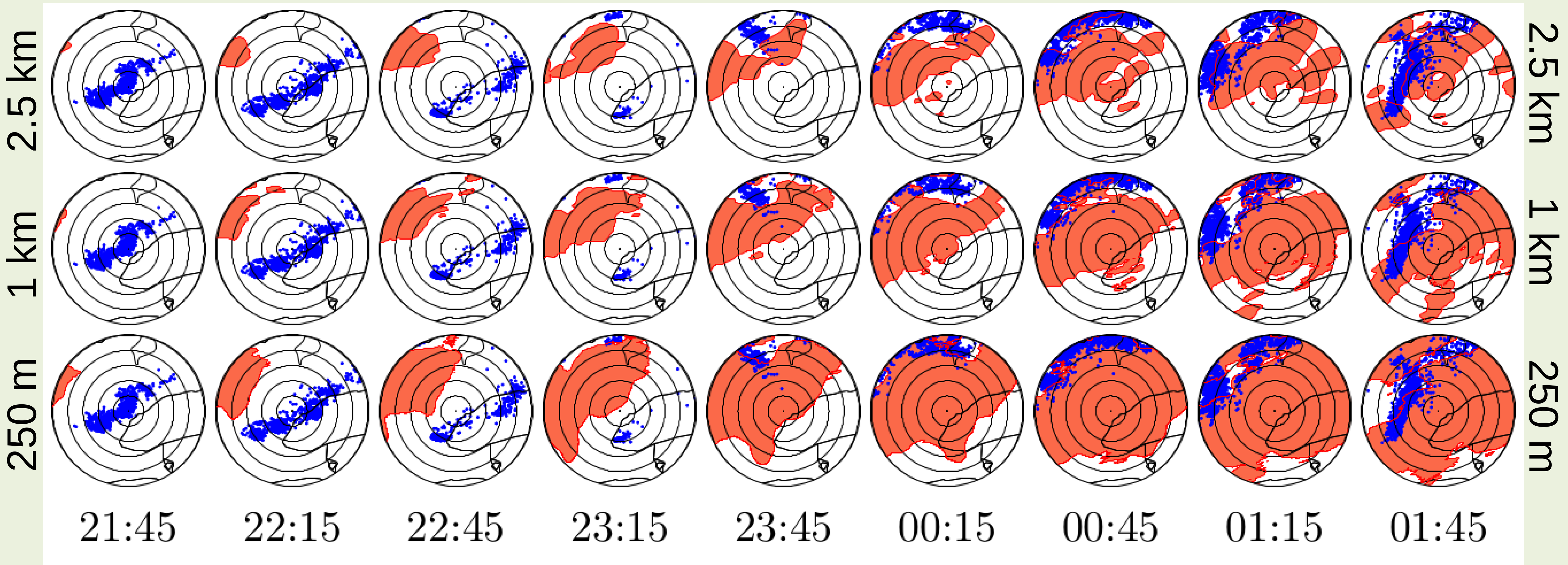
User location at time t — Binary forecast at time t — Binary observation at time t

1) Worst Over-forecast distance Difference:
$\mathbf{WOD} = \max_t (d_O(t) - d_F(t - \Delta T))$;
2) Worst Under-forecast distance Difference:
$\mathbf{WUD} = -\min_t (d_O(t) - d_F(t - \Delta T))$;

Optimal time-lag

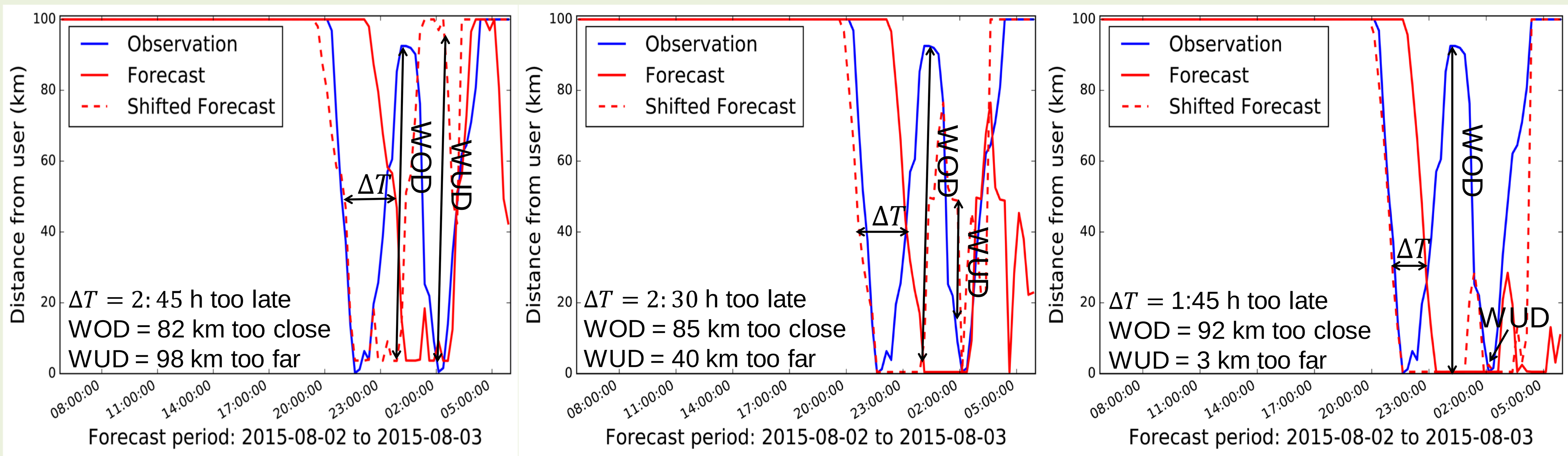
Demonstration

User-centric maps of forecasts vs observations for three different model resolutions for run of 2015/08/02 at 06:00 UTC



Blue: Southern Ontario Lightning Mapping Array (SOLMA) **[2]** flashes.
Red: Lightning parametrization **[3]** of the Canadian High Resolution Deterministic Prediction System (HRDPS) **[4]** for at least one flash per 15 minutes.

- User located at Pearson International Airport near Toronto, Canada.
- GDT with **q = −10**.
- If the distance is greater than **c = 100 km**, we assign 100 km as the distance.
- Time-lag between forecasts and observations estimated by minimizing the mean absolute distance for a lag of less than $\Delta T_{\max} = 3$ **h**.



2.5 km — 1 km — 250 m

Interpretation

Similarly to **[5]**, we can interpret $d_O(t)$ & $d_F(t)$ as a generalization of a 2x2 contingency table:.
- $d_O > 0$ & $d_F = 0$ ⇒ False Alarm
- $d_F > 0$ & $d_O = 0$ ⇒ Miss
- $d_F = 0$ & $d_O = 0$ ⇒ Hit
- $d_F > 0$ & $d_O > 0$ ⇒ Correct Negative

This is extended for *near misses*, i.e. $0 < d_O < c$:
- If $d_O < d_F$, then under-forecasting
- If $d_F < d_O$, then over-forecasting

This is summarized in WOD and WUD:
- If WOD > WUD, then over-forecasting bias
- If WOD < WUD, then under-forecasting bias

A safety margin for spatio-temporal forecasting error can be derived from seasonal verification scores to help decision making.

Advantages

- **Originality:**
  (i) The point-to-set distance is computed via the GDT instead of the classical distance transform.
  (ii) A temporal shift correction ΔT is applied before computing the spatial distances
  (iii) For WOD and WUD, the maximum is taken temporally instead of spatially.
- **Robustness and resilience to outliers**
- **Resolution independence**
- **Intuitiveness and simplicity**
- **Ease of computing**
- **Unique minimizer:**

$$\Delta T = \mathrm{WOD} = \mathrm{WUD} = 0 \Leftrightarrow d_F(t) \equiv d_O(t)$$

References

[1] Brunet, D. and Sills, D., "A Generalized Distance Transform: Theory and Applications to Weather Analysis and Forecasting", IEEE Transactions on Geosciences and Remote Sensing, 2016.
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[3] McCaul et al. "Forecasting lightning threat using cloud-resolving model simulations," Weather and Forecasting, 2009.
[4] Mailhot et al., "An experimental high-resolution forecast system during the Vancouver 2010 Winter Olympics and Paralympic Games," Pure and Applied Geophysics, 2012.
[5] Gilleland, "A new characterization in the spatial verification framework for false alarms, misses, and overall patterns," Weather and Forecasting, 2017.